Name ____ Date ______________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At $1:00$ p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at $1:28$ p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.

   a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at $2:00$ a.m. as at noon?) Explain how you arrived at your estimate.

   b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely $12:00$ p.m.? Explain your answer.

   c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

   a. What quantities could the following expressions represent in terms of the problem context?

   $m + n$

   $mB + nK$

   $$\frac{mB + nK}{m + n}$$

   b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

   $$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

   and explain which expression must be greater using those interpretations.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x+3)(3x+8)-3x(x+3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1+((2+3)\cdot 4)$ is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + \left((b + c) \cdot d\right)$ shows the replacements for the example given.

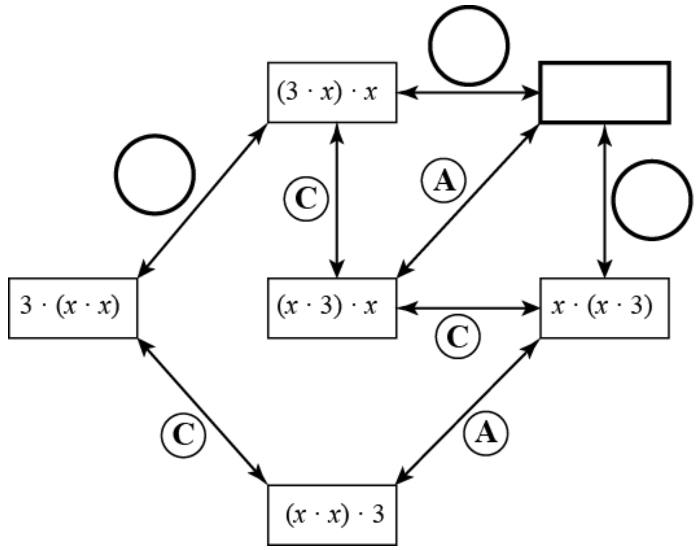
Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

  (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for a, b, c, and d into each expression, the expressions evaluate to **different numbers**, and

  (2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



   a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
   b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10, he says, is 28,130. He doesn't understand why this "rule" is true.
   a. What is the product of the polynomial, $2x^3 + 8x^2 + x + 3$, times the polynomial, x?

   b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

8.

a. Find the following products:

i. $(x-1)(x+1)$

ii. $(x-1)(x^2 + x + 1)$

iii. $(x-1)(x^3 + x^2 + x + 1)$

iv. $(x-1)(x^4 + x^3 + x^2 + x + 1)$

v. $(x-1)(x^n + x^{n-1} + \cdots + x^3 + x^2 + x + 1)$

b. Substitute $x = 10$ into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.

c. If we substituted $x = 10$ into the product $(x-2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?

d. Multiply $(x-2)$ and $(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$, and express your answer in standard form.

Substitute $x=10$ into your answer, and see if you obtain the same result that you obtained in part (c).

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x=10$, multiplying by "$x-9$" is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$.

ii. Put $x=10$ into your answer.

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x=10$?

iii. Was Francois right?

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**a** **N-Q.A.1** **N-Q.A.2**	Student was unable to respond to question. <u>OR</u> Student provided a minimal attempt to create an incorrect graph.	Student created a graph that reflects something related to the problem, but the axes did not depict the correct units of distance from the house on the y-axis and a measurement of time on the x-axis, or the graph indicated significant errors in calculations or reasoning.	Student created axes that depict distance from the house on the y-axis and some measurement of time on the x-axis, and the graph represents a reflection of what occurred but with errors in calculations, missing or erroneous axis labels, or choice of units that makes the graph difficult to obtain information from.	Student created and labeled the y-axis to represent distance from the house in miles and an x-axis to represent time (in minutes past 1:00 p.m.) and created a graph based on solid reasoning and correct calculations.
	b **N-Q.A.1**	Student answered incorrectly with no evidence of reasoning to support the answer. <u>OR</u> Student left item blank.	Student answered incorrectly but demonstrated some reasoning in explaining the answer.	Student answered 1:21 p.m. but did not either refer to a correct graph or provide sound reasoning to support the answer. <u>OR</u> Student answered incorrectly because either the graph in part (a) was incorrect	Student answered 1:21 p.m. and either referred to a correct graph from part (a) or provided reasoning and calculations to explain the answer.

				and the graph was referenced or because a minor calculation error was made but sound reasoning was used.	
	c **N-Q.A.1**	Student answered incorrectly with no evidence of reasoning to support the answer. OR Student left item blank.	Student answered incorrectly but demonstrated some reasoning in explaining the answer.	Student answered 12 miles but did not either refer to the work in part (a) or provide sound reasoning in support of the answer. OR Student answered incorrectly because either the work in part (a) was referenced, but the work was incorrect or because a minor calculation error was made but sound reasoning was used.	Student answered 12 miles and either referenced correct work from part (a) or provided reasoning and calculations to support the answer.
2	**a** **N-Q.A.3**	Student left the question blank. OR Student provided an answer that reflected no or very little reasoning.	Student either began with an assumption that was not based on the evidence of water being used at a rate of approximately 10 liters/second at noon. OR Student used poor reasoning in extending that reading to consider total use across 24 hours.	Student answered beginning with the idea that water was being used at a rate of approximately 10 liters/second at noon but made an error in the calculations to extend and combine that rate to consider usage across 24 hours. OR Student did not defend the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.	Student answered beginning with the idea that water was being used at a rate of approximately 10 liters/second at noon and made correct calculations to extend and combine that rate to consider usage across 24 hours. AND Student defended the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.

	b **N-Q.A.3**	Student left the question blank. OR Student provided an answer that reflected no or very little reasoning.	Student provided an answer that is outside of the range from "to the nearest ten" to "to the nearest hundred". OR Student provided an answer that is within the range but is not supported by an explanation.	Student answer ranged from "to the nearest ten" to "to the nearest hundred" but was not well supported by sound reasoning. OR Student answer contained an error in the way the explanation was written, even if it was clear what the student meant to say.	Student answer ranged from "to the nearest ten" to "to the nearest hundred" and was supported by correct reasoning that is expressed accurately.
	c **N-Q.A.3**	Student left the question blank. OR Student provided an answer that reflected no or very little reasoning.	Student answer was not in the range of 6 to 48 checks but provided some reasoning to justify the choice. OR Student answer was in that range, perhaps written in the form of "every x minutes" or "every x hours" but was not supported by an explanation with solid reasoning.	Student answer was in the range of 6 to 48 checks but was only given in the form of x checks per minute or x checks per hour; the answer was well supported by a written explanation. OR Student answer was given in terms of number of checks but was not well supported by a written explanation.	Student answer was in the range of 6 to 48 checks, and student provided solid reasoning to support the answer.
3	**a** **A-SSE.A.1a** **A-SSE.A.1b**	Student either did not answer. OR Student answered incorrectly for all three expressions.	Student answered one or two of the three correctly but left the other one blank or made a gross error in describing what it represented.	Student answered two of the three correctly and made a reasonable attempt at describing what the other one represented.	Student answered all three correctly.
	b **A-SSE.A.1a** **A-SSE.A.1b**	Student either did not answer. OR Student answered incorrectly for all three parts of the question.	Student understood that the expressions represented a portion of the orders for each color but mis-assigned the colors and/or incorrectly	Student understood that the expressions represented a portion of the orders for each color and correctly determined which one would be larger	Student understood that the expressions represented a portion of the orders for each color, correctly determined which one would be larger,

			determined which one would be larger.	but had errors in the way the answer was worded OR did not provide support for why $\frac{n}{m+n}$ would be larger.	and provided a well written explanation for why.
4	**A-SSE.A.1b** **A-SSE.A.2**	Student left the question blank. OR Student was unable to re-write the expression successfully, even by multiplying out the factors first.	Student got to the correct re-written expression of $8x + 24$ but did so by multiplying out the factors first OR did not show the work needed to demonstrate how $8x + 24$ was determined.	Student attempted to use structure to re-write the expression as described, showing the process, but student made errors in the process.	Student correctly used the process described to arrive at $8x + 24$ without multiplying out linear factors and demonstrated the steps for doing so.
5	**a–b** **A-SSE.A.2**	Student was unable to respond to many of the questions. OR Student left several items blank.	Student was only able to come up with one option for part (a) and, therefore, had only partial work for part (b). OR Student answered "Yes" for the question about equivalent expressions.	Student successfully answered part (a) and identified that the expressions created in part (b) were not equivalent, but there were minor errors in the answering of the remaining questions.	Student answered all four parts correctly and completely.
6	**a** **A-SSE.A.2**	Student left at least three items blank. OR Student answered at least three items incorrectly.	Student answered one or two items incorrectly or left one or more items blank.	Student completed circling task correctly and provided a correct ordering of symbols in the box, but the answer did not use parentheses or multiplication dots.	Student completed all four item correctly, including exact placement of parentheses and symbols for the box: $x \cdot (3 \cdot x)$.
	b **A-SSE.A.2**	Student did not complete either proof successfully.	Student attempted both proofs but made minor errors in both. OR Student only completed one proof, with or without errors.	Student attempted both proofs but made an error in one of them.	Student completed both proofs correctly, and the two proofs were different from one another.

7	a A-APR.A.1	Student left the question blank or demonstrated no understanding of multiplication of polynomials.	Student made more than one error in his multiplication but demonstrated some understanding of multiplication of polynomials.	Student made a minor error in the multiplication.	Student multiplied correctly and expressed the resulting polynomial as a sum of monomials.
	b A-APR.A.1	Student left the question blank or did not demonstrate a level of thinking that was higher than what was given in the problem's description of Ahmed's thinking.	Student used language that did not indicate an understanding of base x and/or the place value system. Student may have used language such as shifting or moving.	Student made only minor errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.	Student made no errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.
8	a–c A-APR.A.1	Student showed limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student made multiple errors but showed some understanding of polynomial multiplication. Student may not have combined like terms to present the product as the sum of monomials.	Student made one or two minor errors but demonstrated knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also showed understanding of evaluating a polynomial for the given value of x.	Student completed all products correctly, expressing each as a sum of monomials with like terms collected, and evaluated correctly when x is 10.
	d A-APR.A.1	Student showed limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student made multiple errors but showed some understanding of polynomial multiplication. Student may not have combined like terms to present the product as the sum of monomials. Student may have gotten an incorrect result when evaluating with $x = 10$.	Student made one or two minor errors but demonstrated knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also showed understanding of evaluating a polynomial for the given value of x.	Student correctly multiplied the polynomials and expressed the product as a polynomial in standard form. Student correctly evaluated with a value of 10 and answered "Yes".

	e **A-APR.A.1**	Student was unable to demonstrate an understanding that part (iii) is "No" and/or demonstrated limited or no understanding of polynomial multiplication.	Student may have made some errors as he multiplied the polynomials and expressed the product as a sum of monomials. Student may have made some errors in the calculation of the value of the polynomial when x is 10. Student incorrectly answered part (iii) or applied incorrect reasoning.	Student may have made minor errors in multiplying the polynomials and expressing the product as a sum of monomials. Student may have made minor errors in calculating the value of the polynomial when x is 10. Student explained that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explained that the two expressions are not algebraically equivalent.	Student correctly multiplied the polynomials and expressed the product as a sum of monomials with like terms collected. Student correctly calculated the value of the polynomial when x is 10. Student explained that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explained that the two expressions are not algebraically equivalent.

Name ______________________________ Date ________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At $1{:}00$ p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late, and drives at a constant speed of 30 mph to the grocery store, arriving at $1{:}28$ p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

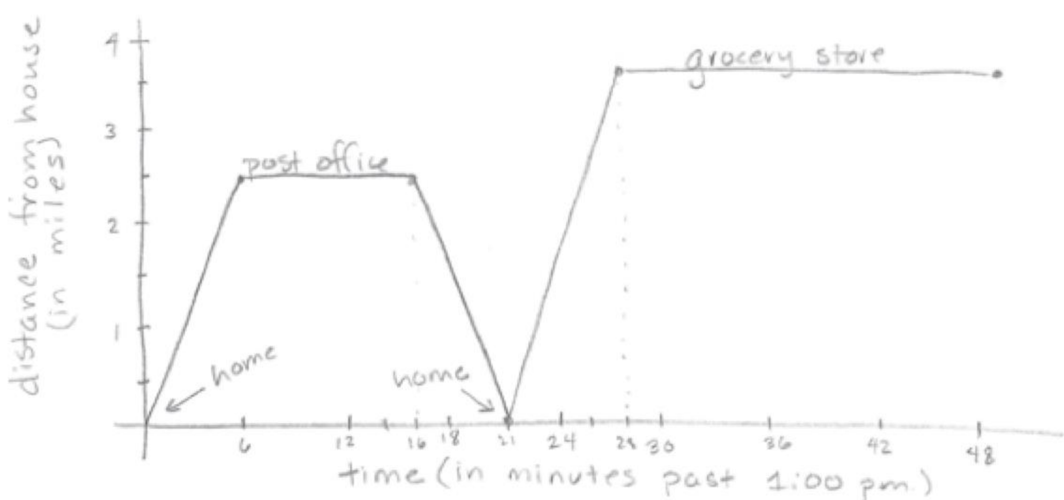
$$25\ \frac{\text{miles}}{\text{hour}} \times 6 \text{ minutes} \times 1\ \frac{\text{hour}}{60 \text{ minutes}} = 2.5 \text{ miles from house to post office}$$

$$30\ \frac{\text{miles}}{\text{hour}} \times 12 \text{ minutes} \times 1\ \frac{\text{hour}}{60 \text{ minutes}} = 6 \text{ miles from post office to store}$$

$$6 \text{ miles} - 2.5 \text{ miles} = 3.5 \text{ miles from home to store}$$

6 miles in 12 minutes is 1 mile in 2 minutes

so 2. 5 miles takes 5 minutes and 3.5 miles takes 7 minutes

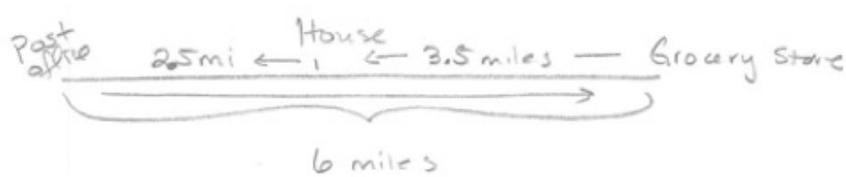


b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

It is 1:21. The graph shows the time as 21 minutes past 1:00 PM. He spent 6 minutes getting to the post office, 10 minutes at the post office, and 5 minutes getting from the post office to the point of passing by his house. You know it took 5 minutes for the last part because he traveled 30 miles per hour and went 2.5 miles. $2.5 \text{ miles} \times \frac{60 \text{ minutes}}{30 \text{ miles}} = 5 \text{ minutes}$

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

12 miles.



$$2.5 \text{ miles} + 6 \text{ miles} + 3.5 \text{ miles} = 12 \text{ miles}$$

You know it is 25 miles from the house to the post office because $25\frac{\text{miles}}{\text{hour}} \times 6 \text{ minutes} \times 1\frac{\text{hour}}{60 \text{ minutes}} = 2.5 \text{ miles.}$

You know it is 6 miles from the post office to the store because $30\frac{\text{miles}}{\text{hour}} \times 12 \text{ minutes} \times 1\frac{\text{hour}}{60 \text{ minutes}} =$ *6 miles.*

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.

a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at $2:00$ a.m. as at noon?) Explain how you arrived at your estimate.

$$10\frac{\text{liters}}{\text{second}} \times 60\frac{\text{seconds}}{\text{minute}} \times 60\frac{\text{minutes}}{\text{hour}} \times 18 \text{ hours} = 648{,}000 \text{ liters}$$

Since water is probably only used from about 5:00 AM to 11:00 PM, I did not multiply by 24 hours, but by 18 hours instead.

b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely $12{:}00$ p.m.? Explain your answer.

It can be reported within ± 10 liters, since he can read the 10's place, but it is changing by a 10 during the second he reads it.

c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

24 checks. Every hour would be good to show the peaks in usage during morning and evening hours from those in the apartments. And it might also show that businesses stop using it after business hours. It would depend on what portion of the building is business vs. apartments.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

a. What quantities could the following expressions represent in terms of the problem context?

$m+n$ – Total number of ink orders over a one-month period.

$mB+nK$ – Total gallons of ink ordered over a one-month period.

$\frac{mB+nK}{m+n}$ – Average number of gallons of ink per order.

b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

$$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

and explain which expression must be greater using those interpretations.

$\frac{m}{m+n}$ is the fraction of orders that are for blue ink.

$\frac{n}{m+n}$ is the fraction of orders that are for black ink.

$\frac{n}{m+n}$ would be bigger, 2 times as big as $\frac{m}{m+n}$ because they ordered twice as many orders for black ink than for blue ink.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x+3)(3x+8)-3x(x+3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

$$\big((3x+8)-3x\big)\cdot(x+3)$$

$$8(x+3)$$

$$8x+24$$

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1+\big((2+3)\cdot 4\big)$ is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

$$(1+2)\cdot(3+4)=21$$

$$\big((2+4)+1\big)\cdot 3=21$$

b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

$$(a + b) \cdot (c + d) = ac + ad + bc + bd$$

$$((b + d) + a) \cdot c = ac + bc + dc$$

Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

(1) Find four different numbers (other than $0, 1, 2, 3, 4$) that when substituted for $a, b, c,$ and d into each expression, the expressions evaluate to **different numbers**, and

$a = 5$ $b = 10$ $c = 20$ $d = 30$

$$(5 + 10) \cdot (20 + 30) = 750$$

$$((10 + 30) + 5) \cdot 20 = 900$$

(2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

$5, 6, 11, 7$

$(5 + 6) \cdot (11 + 7) = 11 \cdot 18 = 198$

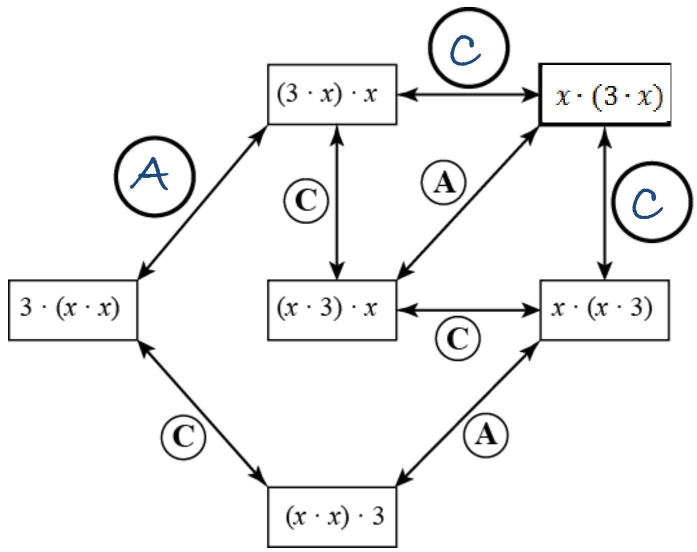
$((6 + 7) + 5) \cdot 11 = 18 \cdot 11 = 198$

$(ac + ad + bc + bd)$ needs to equal $(ac + bc + dc)$;

so, $(ad + bd)$ needs to equal (dc);

so, $(a + b)$ needs to equal c.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.

b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

1. $(x \cdot x) \cdot 3 = x \cdot (x \cdot 3)$ by Associate Property
$x \cdot (x \cdot 3) = x \cdot (3 \cdot x)$ by Commutative Property
$x \cdot (3 \cdot x) = (3 \cdot x) \cdot x$ by Commutative Property

2. $(x \cdot x) \cdot 3 = 3 \cdot (x \cdot x)$ by Commutative Property
$3 \cdot (x \cdot x) = (3 \cdot x) \cdot x$ by Associate Property

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10, he says, is $28{,}130$. He doesn't understand why this "rule" is true.

a. What is the product of the polynomial, $2x^3 + 8x^2 + x + 3$, times the polynomial, x?

$$2x^4 + 8x^3 + x^2 + 3x$$

b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

When you multiply by the same number as the base, it creates a new number where each digit in the original number is now one place-value higher so that there is nothing left (no numbers) to represent the ones' digit, which leads to a trailing "0" in the ones' digit.

8.

a. Find the following products:

i. $(x-1)(x+1)$

$x^2+x-x-1$
x^2-1

ii. $(x-1)\left(x^2+x+1\right)$

$x^3+x^2+x-x^2-x-1$
x^3-1

iii. $(x-1)(x^3+x^2+x+1)$

$x^4+x^3+x^2+x-x^3-x^2-x-1$
x^4-1

iv. $(x-1)(x^4+x^3+x^2+x+1)$

$x^5+x^4+x^3+x^2+x-x^4-x^3-x^2-x-1$
x^5-1

v. $(x-1)(x^n+x^{n-1}+\cdots+x^3+x^2+x+1)$

$x^{n+1}-1$

b. Substitute $x=10$ into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.

i. $(10-1)\cdot(10+1)=(100-1)$
$9\cdot(11)=99$

ii. $(10-1)\cdot(100+10+1)=(1000-1)$
$9\cdot(111)=999$

iii. $(10-1)\cdot(1{,}000+100+10+1)=(10{,}000-1)$
$9\cdot(1{,}111)=9{,}999$

iv. $(10-1)\cdot(10{,}000+1{,}000+100+10+1)=(100{,}000-1)$
$9\cdot(11{,}111)=99{,}999$

c. If we substituted $x=10$ into the product $(x-2)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ and computed the product, what number would result?

$8\cdot(11{,}111{,}111)=88{,}888{,}888$

d. Multiply $(x-2)$ and $(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ and express your answer in standard form.

$$x^8+x^7+x^6+x^5+x^4+x^3+x^2+x-2x^7-2x^6-2x^5-2x^4-2x^3-2x^2-2x-2$$
$$x^8-x^7-x^6-x^5-x^4-x^3-x^2-x-2$$

Substitute $x=10$ into your answer and see if you obtain the same result as you obtained in part (c).

$10^8-10^7-10^6-10^5-10^4-10^3-10^2-10-2=88{,}888{,}888$. Yes, I get the same answer.

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x=10$, multiplying by "$x-9$" is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$.

$$x^8-8x^7-8x^6-8x^5-8x^4-8x^3-8x^2-8x-9$$

ii. Put $x=10$ into your answer.

$$100{,}000{,}000-80{,}000{,}000-8{,}000{,}000-800{,}000-80{,}000-8{,}000-800-80-9$$

$$100{,}000{,}000-88{,}888{,}889=11{,}111{,}111$$

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x=10$?

Yes.

iii. Was Francois right?

No, just because it is true when x is 10, doesn't make it true for all real x. The two expressions are not algebraically equivalent.