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The Concept of Congruence

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[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 8 • Module 2

The Concept of Congruence

OVERVIEW

In this module, students learn about translations, reflections, and rotations in the plane and, more importantly, how to use them to precisely define the concept of *congruence*. Up to this point, "congruence" has been taken to mean, intuitively, "same size and same shape." Because this module begins with a serious study of geometry, this intuitive definition must be replaced by a precise definition. This module is a first step; its goal is to provide the needed intuitive background for the precise definitions that are introduced in this module for the first time.

Translations, reflections, and rotations are examples of *rigid motions*, which are, intuitively, rules of moving points in the plane in such a way that preserves distance. For the sake of brevity, these three rigid motions will be referred to exclusively as the *basic rigid motions*. Initially, the exploration of these basic rigid motions is done via hands-on activities using an overhead projector transparency, but with the availability of geometry software, the use of technology in this learning environment is inevitable, and some general guidelines for this usage will be laid out at the end of Lesson 2. What needs to be emphasized is that the importance of these basic rigid motions lies not in the fun activities they bring but in the *mathematical* purpose they serve in clarifying the meaning of congruence.

Throughout Topic A, on the definitions and properties of the basic rigid motions, students verify experimentally their basic properties and, when feasible, deepen their understanding of these properties using reasoning. In particular, what students learned in Grade 4 about angles and angle measurement (**4.MD.C.5**) will be put to good use here. They learn that the basic rigid motions preserve angle measurements, as well as segment lengths.

Topic B is a critical foundation to the understanding of congruence. All the lessons of Topic B demonstrate to students the ability to sequence various combinations of rigid motions while maintaining the basic properties of individual rigid motions. Lesson 7 begins this work with a sequence of translations. Students verify experimentally that a sequence of translations have the same properties as a single translation. Lessons 8 and 9 demonstrate sequences of reflections and translations and sequences of rotations. The concept of sequencing a combination of all three rigid motions is introduced in Lesson 10; this paves the way for the study of congruence in the next topic.

In Topic C, which introduces the definition and properties of congruence, students learn that congruence is just a sequence of basic rigid motions. The fundamental properties shared by all the basic rigid motions are then inherited by congruence: Congruence moves lines to lines and angles to angles, and it is both distance- and angle-preserving (Lesson 11). In Grade 7, students used facts about supplementary, complementary, vertical, and adjacent angles to find the measures of unknown angles (**7.G.B.5**). This module extends that knowledge to angle relationships that are formed when two parallel lines are cut by a transversal. In Topic C, on angle relationships related to parallel lines, students learn that pairs of angles are congruent because they are angles that have been translated along a transversal, rotated around a point, or reflected across a line.

Students use this knowledge of angle relationships in Lessons 13 and 14 to show why a triangle has a sum of interior angles equal to 180° and why the measure of each exterior angle of a triangle is the sum of the measures of the two remote interior angles of the triangle.

Optional Topic D introduces the Pythagorean theorem. Students are shown the "square within a square" proof of the Pythagorean theorem. The proof uses concepts learned in previous topics of the module, i.e., the concept of congruence and concepts related to degrees of angles. Students begin the work of finding the length of a leg or hypotenuse of a right triangle using $a^2 + b^2 = c^2$. Note that this topic will not be assessed until Module 7.

Focus Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1   Verify experimentally the properties of rotations, reflections, and translations:

   a.   Lines are taken to lines, and line segments to line segments of the same length.

   b.   Angles are taken to angles of the same measure.

   c.   Parallel lines are taken to parallel lines.

8.G.A.2   Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.5   Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

8.G.B.6   Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7   Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Foundational Standards

Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5   Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

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a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** This module is rich with notation that requires students to decontextualize and contextualize throughout. Students work with figures and their transformed images using symbolic representations and need to attend to the meaning of the symbolic notation to contextualize problems. Students use facts learned about rigid motions in order to make sense of problems involving congruence.

MP.3 **Construct viable arguments and critique the reasoning of others.** Throughout this module, students construct arguments around the properties of rigid motions. Students make assumptions about parallel and perpendicular lines and use properties of rigid motions to directly or indirectly prove their assumptions. Students use definitions to describe a sequence of rigid motions to prove or disprove congruence. Students build a logical progression of statements to show relationships between angles of parallel lines cut by a transversal, the angle sum of triangles, and properties of polygons like rectangles and parallelograms.

MP.5 **Use appropriate tools strategically**. This module relies on students' fundamental understanding of rigid motions. As a means to this end, students use a variety of tools but none as important as an overhead transparency. Students verify experimentally the properties of rigid motions using physical models and transparencies. Students use

transparencies when learning about translation, rotation, reflection, and congruence in general. Students determine when they need to use the transparency as a tool to justify conjectures or when critiquing the reasoning of others.

MP.6 **Attend to precision**. This module begins with precise definitions related to transformations and statements about transformations being distance- and angle-preserving. Students are expected to attend to the precision of these definitions and statements consistently and appropriately as they communicate with others. Students describe sequences of motions precisely and carefully label diagrams so that there is clarity about figures and their transformed images. Students attend to precision in their verbal and written descriptions of rays, segments, points, angles, and transformations in general.

Terminology

New or Recently Introduced Terms

- **Transformation** (A *transformation* is a rule, to be denoted by F, that assigns each point P of the plane a unique point which is denoted by $F(P)$.)
- **Basic Rigid Motion** (A *basic rigid motion* is a rotation, reflection, or translation of the plane.
  - Basic rigid motions are examples of transformations. Given a transformation, the image of a point A is the point the transformation maps the point A to in the plane.)
- **Translation** (A *translation* is a basic rigid motion that moves a figure along a given vector.)
- **Rotation** (A *rotation* is a basic rigid motion that moves a figure around a point, d degrees.)
- **Reflection** (A *reflection* is a basic rigid motion that moves a figure across a line.)
- **Image of a point, image of a figure** (*Image* refers to the location of a point or figure after it has been transformed.)
- **Sequence (Composition) of Transformations** (A *sequence of transformations* is more than one transformation. Given transformations G and F, $G \circ F$ is called the composition of F and G.)
- **Vector** (A Euclidean *vector* (or directed segment) $\overrightarrow{AB}$ is the line segment AB together with a direction given by connecting an initial point A to a terminal point B.)
- **Congruence** (A *congruence* is a sequence of basic rigid motions (rotations, reflections, translations) of the plane.)
- **Transversal** (Given a pair of lines L and M in a plane, a third line T is a *transversal* if it intersects L at a single point and intersects M at a single but different point.)

Familiar Terms and Symbols[2]

- Ray, line, line segment, angle
- Parallel and perpendicular lines

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[2] These are terms and symbols students have seen previously.

- Supplementary, complementary, vertical, and adjacent angles
- Triangle, quadrilateral
- Area and perimeter

Suggested Tools and Representations

- Transparency or patty paper
- Wet or dry erase markers for use with transparency
- Optional: geometry software
- Composition of Rigid Motions: http://youtu.be/O2XPy3ZLU7Y
- ASA: http://www.youtube.com/watch?v=-yIZdenw5U4

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	8.G.A.1
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	8.G.A.2, 8.G.A.5