Name ____ Date ____________________

Lesson 1: Successive Differences in Polynomials

Exit Ticket

1. What type of relationship is indicated by the following set of ordered pairs? Explain how you know.

x	y
0	0
1	2
2	10
3	24
4	44

2. Find an equation that all ordered pairs above satisfy.

Name ____ Date ____________________

Lesson 2: The Multiplication of Polynomials

Exit Ticket

Multiply $(x-1)(x^3+4x^2+4x-1)$ and combine like terms. Explain how you reached your answer.

Name ____ Date ____________________

Lesson 3: The Division of Polynomials

Exit Ticket

Find the quotient. Justify your answer.

$$\frac{x^5 + 2x^4 - 7x^2 - 19x + 15}{x^2 + 2x + 5}$$

Name ____ Date ____________________

Lesson 4: Comparing Methods—Long Division, Again?

Exit Ticket

Write a note to a friend explaining how to use long division to find the quotient.

$$\frac{2x^2 - 3x - 5}{x + 1}$$

Name ____ Date ____________________

Lesson 5: Putting It All Together

Exit Ticket

Jenny thinks that the expression below is equal to $x^2 - 4$. If you agree, show that she is correct. If you disagree, show that she is wrong by re-writing this expression as a polynomial in standard form.

$$\frac{(x-2)^3}{x-2}$$

Name ______________________________ Date ____________________

Lesson 6: Dividing by $x - a$ and by $x + a$

Exit Ticket

Compute each quotient using the identities you discovered in this lesson. Then, write the problem as a product. An example is shown.

QUOTIENT	PRODUCT
$\frac{x^2-9}{x+3} = x - 3$	$x^2 - 9 = (x+3)(x-3)$

1. $\frac{x^4-16}{x-2}$

2. $\frac{x^3+1000}{x+10}$

3. $\frac{x^5-1}{x-1}$

Name ____ Date ____________________

Lesson 7: Mental Math

Exit Ticket

1. Explain how to use the patterns in this lesson to quickly compute $(57)(43)$.

2. Jessica believes that $10^3 - 1$ is divisible by 9. Use your work from this lesson to support or refute her claim.

Name ____ Date ____________________

Lesson 8: The Power of Algebra—Finding Primes

Exit Ticket

Express the prime number 31 in the form $2^p - 1$ where p is a prime number and as a difference of two perfect squares using the identity $(a+b)(a-b) = a^2 - b^2$.

Name ____ Date ____________________

Lesson 9: Radicals and Conjugates

Exit Ticket

1. Rewrite each of the following radicals as a rational number or in simplest radical form.
   a. $\sqrt{49}$
   b. $\sqrt[3]{40}$
   c. $\sqrt{242}$

2. Find the conjugate of each of the following radical expressions.
   a. $\sqrt{5}+\sqrt{11}$
   b. $9-\sqrt{11}$
   c. $\sqrt[3]{3}+1.5$

3. Rewrite each of the following expressions as a rational number or in simplest radical form.
   a. $\sqrt{3}\left(\sqrt{3}-1\right)$
   b. $\left(5+\sqrt{3}\right)^2$
   c. $(10+\sqrt{11})(10-\sqrt{11})$

Name ____ Date ____________________

Lesson 10: The Power of Algebra—Finding Pythagorean Triples

Exit Ticket

Generate six Pythagorean triples using any method discussed during class. Explain each method you use.

Name ______________________________ Date ______________

Lesson 11: The Special Role of Zero in Factoring

Exit Ticket

A polynomial function p can be factored into seven factors: $(x-3)$, $(x+1)$, and 5 factors of $(x-2)$. What are its zeros with multiplicity, and what is the degree of the polynomial? Explain how you know.

Name ____ Date ____________________

Lesson 12: Overcoming Obstacles in Factoring

Exit Ticket

Solve the following equation and explain your solution method.

$$x^3 + 7x^2 - x - 7 = 0$$

Name ______________________________ Date ________________

Lesson 13: Mastering Factoring

Exit Ticket

1. Factor the following expression and verify that the factored expression is equivalent to the original.

$$4x^2 - 9a^6$$

2. Factor the following expression and verify that the factored expression is equivalent to the original

$$16x^2 - 8x - 3$$

Name ____ Date ____________________

Lesson 14: Graphing Factored Polynomials

Exit Ticket

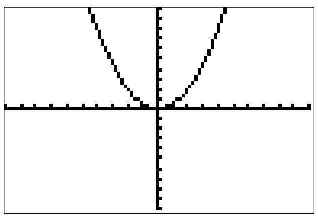
Sketch a graph of the function $f(x) = x^3 + x^2 - 4x - 4$ on the interval $[-2, 2]$ by finding the zeros and determining the sign of the function between zeros. Explain how the structure of the equation helps guide your sketch.

Name ______________________________ Date ______________

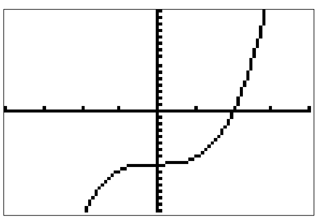
Lesson 15: Structure in Graphs of Polynomial Functions

Exit Ticket

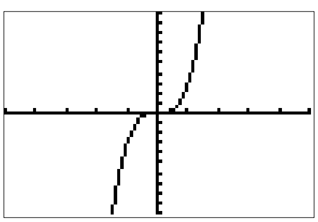
Without using a graphing utility, match each graph below in column 1 with the function in column 2 that it represents.

a. 

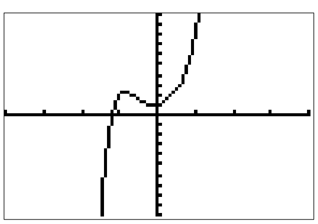
1. $y = 3x^3$

b. 

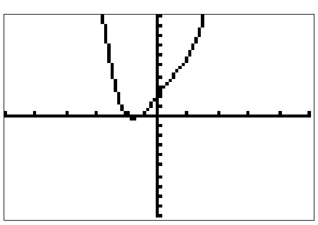
2. $y = \frac{1}{2}x^2$

c. 

3. $y = x^3 - 8$

d. 

4. $y = x^4 - x^3 + 4x + 2$

e. 

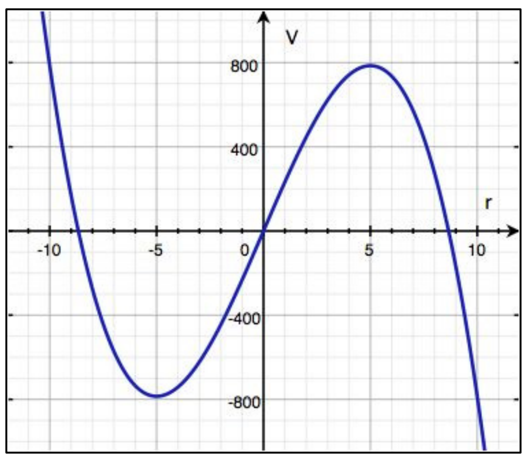
5. $y = 3x^5 - x^3 + 4x + 2$

Name ____ Date ____________________

Lesson 16: Modeling with Polynomials—An Introduction

Exit Ticket

Jeannie wishes to construct a cylinder closed at both ends. The figure below shows the graph of a cubic polynomial function used to model the volume of the cylinder as a function of the radius if the cylinder is constructed using 150π cm^2 of material. Use the graph to answer the questions below. Estimate values to the nearest half unit on the horizontal axis and to the nearest 50 units on the vertical axis.



1. What is the domain of the volume function? Explain.

2. What is the most volume that Jeannie's cylinder can enclose?

3. What radius yields the maximum volume?

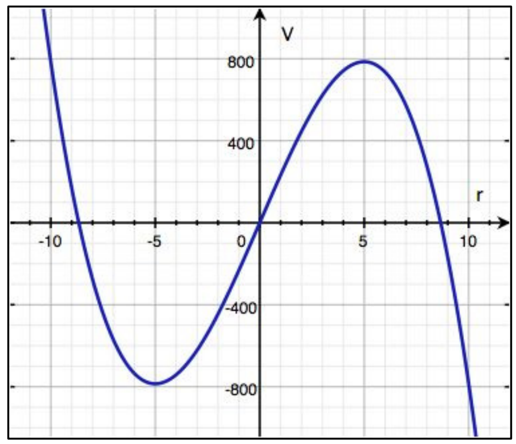
4. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Calculate the height of the cylinder that maximizes the volume.

Name ____ Date ____________________

Lesson 17: Modeling with Polynomials—An Introduction

Exit Ticket

Jeannie wishes to construct a cylinder closed at both ends. The figure at right shows the graph of a cubic polynomial function used to model the volume of the cylinder as a function of the radius if the cylinder is constructed using $150\pi\ \text{cm}^3$ of material. Use the graph to answer the questions below. Estimate values to the nearest half unit on the horizontal axis and to the nearest 50 units on the vertical axis.



1. What are the zeros of the function?

2. What are the relative maximum and the relative maximum values of the function?

3. The equation of this function is $V(r) = c(r^3 - 72.25r)$ for some real number c. Find the value of c so that this formula fits the graph.

4. Use the graph to estimate the volume of the cylinder with $r = 2\text{ cm}$.

5. Use your formula to find the volume of the cylinder when $r = 2\text{ cm}$. How close is the value from the formula to the value on the graph?

Name ____ Date ____________________

Lesson 18: Overcoming a Second Obstacle in Factoring—What If There Is a Remainder?

Exit Ticket

1. Find the quotient of $\frac{x-6}{x-8}$ by inspection.

2. Find the quotient of $\frac{9x^3-12x^2+4}{x-2}$ by using either long division or the reverse tabular method.

Name ___________________________________ Date ____________________

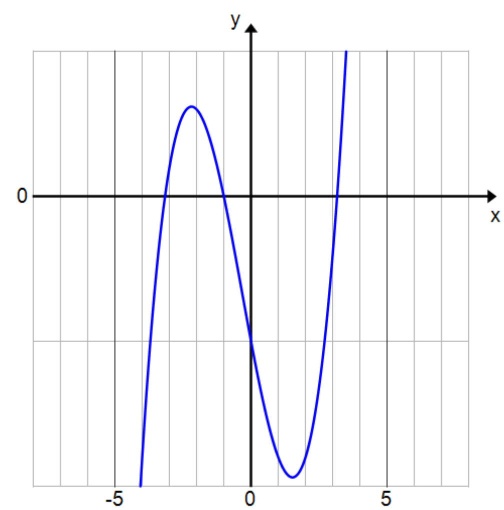
Lesson 19: The Remainder Theorem

Exit Ticket

Consider the polynomial $P(x) = x^3 + x^2 - 10x - 10$.

1. Is $x + 1$ one of the factors of P? Explain.

2. The graph shown has x-intercepts at $\sqrt{10}$, -1, and $-\sqrt{10}$. Could this be the graph of $P(x) = x^3 + x^2 - 10x - 10$? Explain how you know.



Name ____ Date ____________________

Lesson 20: Modeling Riverbeds with Polynomials

Exit Ticket

Use the Remainder Theorem to find a quadratic polynomial P so that $P(1) = 5$, $P(2) = 12$, and $P(3) = 25$. Give your answer in standard form.

Name ____ Date ____________________

Lesson 21: Modeling Riverbeds with Polynomials

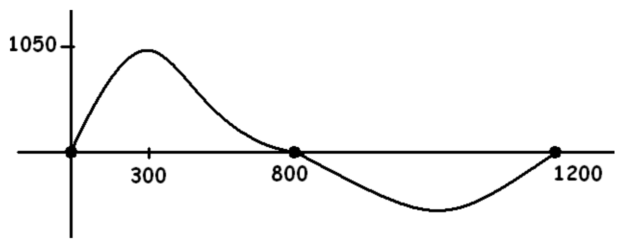
Exit Ticket

Explain the process you used to estimate the volumetric flow of the river, from accumulating the data to calculating the flow of water.

Name ____ Date ____________________

1. Geographers sit at a café discussing their field work site, which is a hill and a neighboring riverbed. The hill is approximately $1{,}050$ ft. high, 800 ft. wide, with peak about 300 ft. east of the western base of the hill. The river is about 400 ft. wide. They know the river is shallow, no more than about 20 ft. deep.

   They make the following crude sketch on a napkin, placing the profile of the hill and riverbed on a coordinate system with the horizontal axis representing ground level.



   The geographers do not have any computing tools with them at the café, so they decide to use pen and paper to compute a cubic polynomial that approximates this profile of the hill and riverbed.

   a. Using only a pencil and paper, write a cubic polynomial function H that could represent the curve shown (here, x represents the distance, in feet, along the horizontal axis from the western base of the hill, and $H(x)$ is the height, in feet, of the land at that distance from the western base). Be sure that your formula satisfies $H(300) = 1050$.

b. For the sake of convenience, the geographers make the assumption that the deepest point of the river is halfway across the river (recall that the river is no more than 20-ft. deep). Under this assumption, would a cubic polynomial provide a suitable model for this hill and riverbed? Explain.

2. Luke notices that by taking any three consecutive integers, multiplying them together, and adding the middle number to the result, the answer always seems to be the middle number cubed.

For example:

$$3 \times 4 \times 5 + 4 = 64 = 4^3$$
$$4 \times 5 \times 6 + 5 = 125 = 5^3$$
$$9 \times 10 \times 11 + 10 = 1000 = 10^3$$

a. To prove his observation, Luke writes $(n+1)(n+2)(n+3)+(n+2)$. What answer is he hoping to show this expression equals?

b. Lulu, upon hearing of Luke's observation, writes her own version with n as the middle number. What does her formula look like?

c. Use Lulu's expression to prove that adding the middle number to the product of any three consecutive numbers is sure to equal that middle number cubed.

3. A cookie company packages its cookies in rectangular prism boxes designed with square bases that have both a length and width of 4 in. less than the height of the box.

   a. Write a polynomial that represents the volume of a box with height x inches.

   b. Find the dimensions of the box if its volume is 128 cubic inches.

c. After solving this problem, Juan was very clever and invented the following strange question:

A building, in the shape of a rectangular prism with a square base, has on its top a radio tower. The building is 25 *times as tall as the tower, and the side-length of the base of the building is* 100 *ft. less than the height of the building. If the building has a volume of* 2*-million cubic feet, how tall is the tower?*

Solve Juan's problem.

Name ____ Date ____________________

Lesson 22: Equivalent Rational Expressions

Exit Ticket

1. Find an equivalent rational expression in lowest terms, and identify the value(s) of the variables that must be excluded to prevent division by zero.

$$\frac{x^2 - 7x + 12}{6 - 5x + x^2}$$

2. Determine whether or not the rational expressions $\frac{x+4}{(x+2)(x-3)}$ and $\frac{x^2+5x+4}{(x+1)(x+2)(x-3)}$ are equivalent for $x \neq -1$, $x \neq -2$, and $x \neq 3$. Explain how you know.

Name ______________________________ Date ______________

Lesson 23: Comparing Rational Expressions

Exit Ticket

Use the specified methods to compare the following rational expressions: $\frac{x+1}{x^2}$ and $\frac{1}{x}$ for $x > 0$.

1. Fill out the table of values.

x	$\dfrac{x+1}{x^2}$	$\dfrac{1}{x}$
1		
10		
25		
50		
100		
500		

2. Graph $y = \frac{x+1}{x^2}$ and $y = \frac{1}{x}$ for positive values of x.

3. Find the common denominator and compare numerators for positive values of x.

Name ____ Date ____________________

Lesson 24: Multiplying and Dividing Rational Expressions

Exit Ticket

Perform the indicated operations and reduce to lowest terms.

1. $\dfrac{x-2}{x^2+x-2} \cdot \dfrac{x^2-3x+2}{x+2}$

2. $\dfrac{\left(\dfrac{x-2}{x^2+x-2}\right)}{\left(\dfrac{x^2-3x+2}{x+2}\right)}$

Name ______________________________ Date ______________

Lesson 25: Adding and Subtracting Rational Expressions

Exit Ticket

Perform the indicated operation.

1. $\frac{3}{a+2} + \frac{4}{a-5}$

2. $\frac{4r}{r+3} - \frac{5}{r}$

Name ____ Date ____________________

Lesson 26: Solving Rational Equations

Exit Ticket

Find all solutions to the following equation. If there are any extraneous solutions, identify them and explain why they are extraneous.

$$\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b}{b^2-9}$$

Name ____ Date ____________________

Lesson 27: Word Problems Leading to Rational Equations

Exit Ticket

Bob can paint a fence in 5 hours, and working with Jen, the two of them painted the fence in 2 hours. How long would it have taken Jen to paint the fence alone?

Name ____ Date ____________________

Lesson 28: A Focus on Square Roots

Exit Ticket

Consider the radical equation $3\sqrt{6-x}+4=-8$.

1. Solve the equation. Next to each step, write a description of what is being done.

2. Check the solution.

3. Explain why the calculation in Problem 1 does not produce the solution to the equation.

Name ____ Date ____________________

Lesson 29: Solving Radical Equations

Exit Ticket

1. Solve $\sqrt{2x+15} = x + 6$. Verify the solution(s).

2. Explain why it is necessary to check the solutions to a radical equation.

Name ____ Date ____________________

Lesson 30: Linear Systems in Three Variables

Exit Ticket

For the following system, determine the values of p, q, and r that satisfy all three equations:

$$\begin{aligned} 2p + q - r &= 8 \\ q + r &= 4 \\ p - q \quad &= 2 \end{aligned}$$

Name ____ Date____________________

Lesson 31: Systems of Equations

Exit Ticket

Make and explain a prediction about the nature of the solution to the following system of equations and then solve it.

$$x^2 + y^2 = 25$$
$$4x + 3y = 0$$

Illustrate with a graph. Verify your solution and compare it with your initial prediction.

Name ____ Date ____________________

Lesson 32: Graphing Systems of Equations

Exit Ticket

1. Find the intersection of the two circles

$$x^2 + y^2 - 2x + 4y - 11 = 0$$

and

$$x^2 + y^2 + 4x + 2y - 9 = 0.$$

2. The equations of the two circles in Question 1 can also be written as follows:

$$(x - 1)^2 + (y + 2)^2 = 16$$

and

$$(x + 2)^2 + (y + 1)^2 = 14.$$

Graph the circles and the line joining their points of intersection.

3. Find the distance between the centers of the circles in Questions 1 and 2.

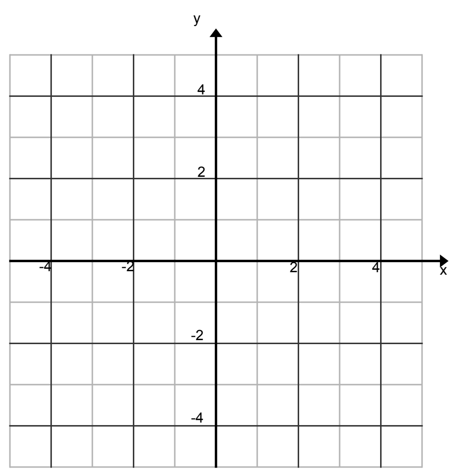
Name ______________________________ Date ______________

Lesson 33: The Definition of a Parabola

Exit Ticket

1. Derive an analytic equation for a parabola whose focus is $(0,4)$ and directrix is the x-axis. Explain how you got your answer.

2. Sketch the parabola from Question 1. Label the focus and directrix.



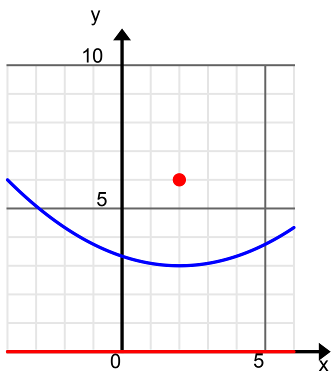
Name ____ Date ____________________

Lesson 34: Are All Parabolas Congruent?

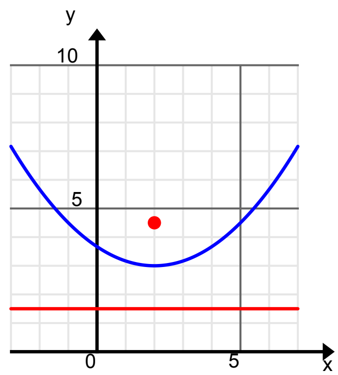
Exit Ticket

Which parabolas shown below are congruent to the parabola that is the graph of the equation $y = \frac{1}{12}x^2$? Explain how you know.

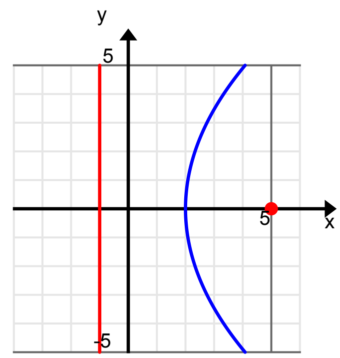
a.



b.



c.



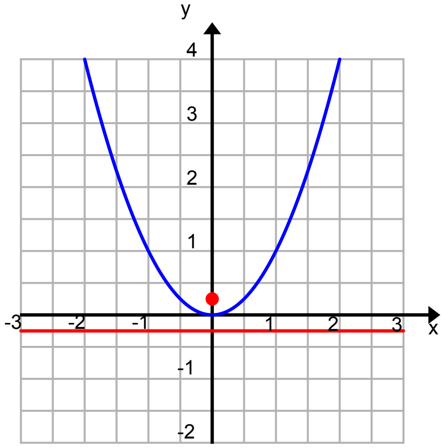
Name ____ Date ____________________

Lesson 35: Are All Parabolas Similar?

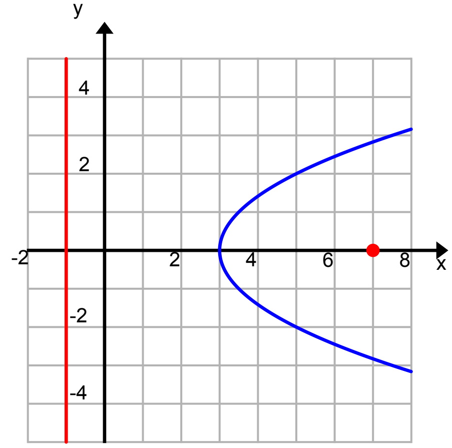
Exit Ticket

1. Describe the sequence of transformations that will transform the parabola P_x into the similar parabola P_y.

Graph of P_x



Graph of P_y



2. Are the two parabolas defined below similar or congruent or both? Justify your reasoning.

Parabola 1: The parabola with a focus of $(0,2)$ and a directrix line of $y = -4$

Parabola 2: The parabola that is the graph of the equation $y = \frac{1}{6}x^2$

Name ____ Date ____________________

Lesson 36: Overcoming a Third Obstacle—What If There Are No Real Number Solutions?

Exit Ticket

Solve the following system of equations or show that it does not have a real solution. Support your answer analytically and graphically.

$$y = x^2 - 4$$
$$y = -(x + 5)$$

Name ____ Date ____________________

Lesson 37: A Surprising Boost from Geometry

Exit Ticket

Express the quantities below in $a + bi$ form, and graph the corresponding points on the complex plane. If you use one set of axes, be sure to label each point appropriately.

$$(1 + i) - (1 - i)$$

$$(1 + i)(1 - i)$$

$$i(2 - i)(1 + 2i)$$

Name ____ Date ____________________

Lesson 38: Complex Numbers as Solutions to Equations

Exit Ticket

Use the discriminant to predict the nature of the solutions to the equation $4x - 3x^2 = 10$. Then, solve the equation.

Name ____ Date ____________________

Lesson 39: Factoring Extended to the Complex Realm

Exit Ticket

1. Solve the quadratic equation $x^2 + 9 = 0$. What are the x-intercepts of the graph of the function $f(x) = x^2 + 9$?

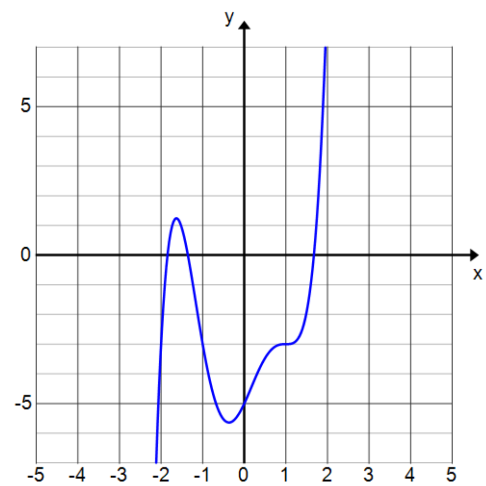
2. Find the solutions to $2x^5 - 5x^3 - 3x = 0$. What are the x-intercepts of the graph of the function $f(x) = 2x^5 - 5x^3 - 3x$?

Name ____ Date ____________________

Lesson 40: Obstacles Resolved—A Surprising Result

Exit Ticket

Consider the degree 5 polynomial function $P(x) = x^5 - 4x^3 + 2x^2 + 3x - 5$, whose graph is shown below. You do not need to factor this polynomial to answer the questions below.



1. How many linear factors is P guaranteed to have? Explain.

2. How many zeros does P have? Explain.

3. How many real zeros does P have? Explain.

4. How many complex zeros does P have? Explain.

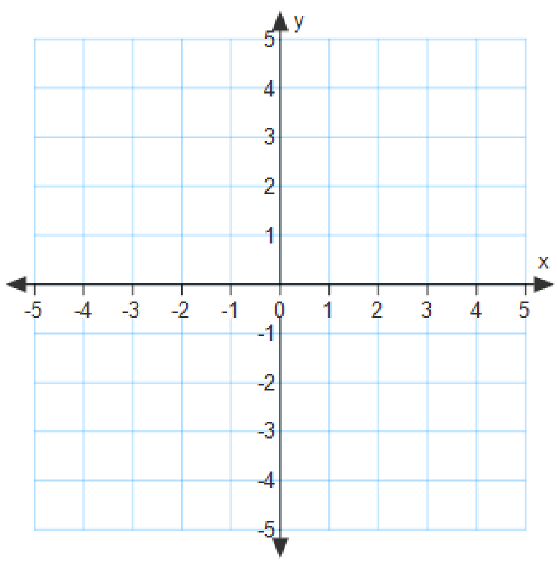
Name ____ Date ______________________

1. A *parabola* is defined as the set of points in the plane that are equidistant from a fixed point (called the *focus* of the parabola) and a fixed line (called the *directrix* of the parabola).

   Consider the parabola with focus point $(1, 1)$ and directrix the horizontal line $y = -3$.

   a. What are the coordinates of the vertex of the parabola?

   b. Plot the focus and draw the directrix on the graph below. Then draw a rough sketch of the parabola.



c. Find the equation of the parabola with this focus and directrix.

d. What is the y-intercept of this parabola?

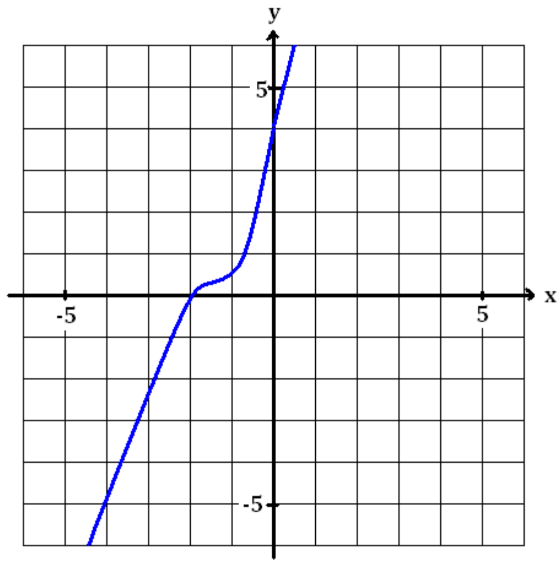
e. Demonstrate that your answer from part (d) is correct by showing that the y-intercept you identified is indeed equidistant from the focus and the directrix.

f. Is the parabola in this question (with focus point $(1, 1)$ and directrix $y = -3$) congruent to a parabola with focus $(2, 3)$ and directrix $y = -1$? Explain.

g. Is the parabola in this question (with focus point $(1, 1)$ and directrix $y = -3$) congruent to the parabola with equation given by $y = x^2$? Explain.

h. Are the two parabolas from part (g) similar? Why or why not?

2. The graph of the polynomial function $f(x) = x^3 + 4x^2 + 6x + 4$ is shown below.



a. Based on the appearance of the graph, what does the real solution to the equation $x^3 + 4x^2 + 6x + 4 = 0$ appear to be? Jiju does not trust the accuracy of the graph. Prove to her algebraically that your answer is in fact a zero of $y = f(x)$.

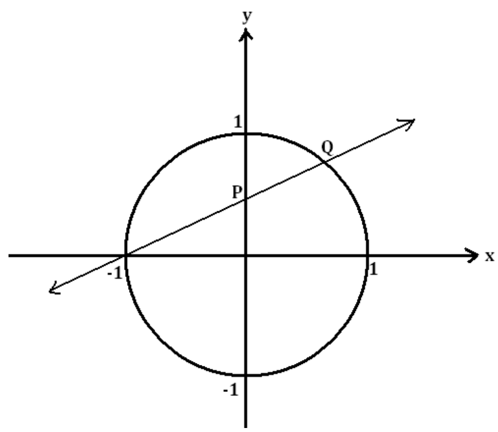
b. Write f as a product of a linear factor and a quadratic factor, each with real-number coefficients.

c. What is the value of $f(10)$? Explain how knowing the linear factor of f establishes that $f(10)$ is a multiple of 12.

d. Find the two complex-number zeros of $y = f(x)$.

e. Write f as a product of three linear factors.

3. A line passes through the points $(-1, 0)$ and $P = (0, t)$ for some real number t and intersects the circle $x^2 + y^2 = 1$ at a point Q different from $(-1, 0)$.



a. If $t = \frac{1}{2}$, so that the point P has coordinates $\left(0, \frac{1}{2}\right)$, find the coordinates of the point Q.

A Pythagorean triple is a set of three positive integers a, b, and c satisfying $a^2 + b^2 = c^2$. For example, setting $a = 3$, $b = 4$, and $c = 5$ gives a Pythagorean triple.

b. Suppose that $\left(\frac{a}{c}, \frac{b}{c}\right)$ is a point with rational-number coordinates lying on the circle $x^2 + y^2 = 1$. Explain why then a, b, and c form a Pythagorean triple.

c. Which Pythagorean triple is associated with the point $Q = \left(\frac{5}{13}, \frac{12}{13}\right)$ on the circle?

d. If $Q = \left(\frac{5}{13}, \frac{12}{13}\right)$, what is the value of t so that the point P has coordinates $(0, t)$?

e. Suppose we set $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, for a real number t. Show that (x, y) is then a point on the circle $x^2 + y^2 = 1$.

f. Set $t = \frac{3}{4}$ in the formulas $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$. Which point on the circle $x^2 + y^2 = 1$ does this give? What is the associated Pythagorean triple?

g. Suppose t is a value greater than 1, $P = (0, t)$, and Q is the point in the second quadrant (different from $(-1, 0)$) at which the line through $(-1, 0)$ and P intersects the circle $x^2 + y^2 = 1$. Find the coordinates of the point Q in terms of t.

4.

a. Write a system of two equations in two variables where one equation is quadratic and the other is linear such that the system has no solution. Explain, using graphs, algebra, and/or words, why the system has no solution.

b. Prove that $x = \sqrt{-5x - 6}$ has no solution.

c. Does the following system of equations have a solution? If so, find one. If not, explain why not.

$$\begin{gathered} 2x + y + z = 4 \\ x - y + 3z = -2 \\ -x + y + z = -2 \end{gathered}$$