Name ______________________________ Date ________________

1. The number of users of social media has increased significantly since the year 2001. In fact, the approximate number of users has tripled each year. It was reported that in 2005 there were 3 million users of social media.

   a. Assuming that the number of users continues to triple each year, for the next three years, determine the number of users in 2006, 2007, and 2008.

   b. Assume the trend in the numbers of users tripling each year was true for all years from 2001 to 2009. Complete the table below using 2005 as year 1 with 3 million as the number of users that year.

Year	-3	-2	-1	0	1	2	3	4	5
# of users in millions					3				

   c. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 2, 3, 4, and 5?

   d. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 0, -1, -2, and -3?

e. Write an equation to represent the number of users in millions, N, for year, t, $t \geq -3$.

f. Using the context of the problem, explain whether or not the formula, $N = 3^t$ would work for finding the number of users in millions in year t, for all $t \leq 0$.

g. Assume the total number of users continues to triple each year after 2009. Determine the number of users in 2012. Given that the world population at the end of 2011 was approximately 7 billion, is this assumption reasonable? Explain your reasoning.

2. Let m be a whole number.

   a. Use the properties of exponents to write an equivalent expression that is a product of unique primes, each raised to an integer power.

$$\frac{6^{21} \cdot 10^{7}}{30^{7}}$$

   b. Use the properties of exponents to prove the following identity:

$$\frac{6^{3m} \cdot 10^{m}}{30^{m}} = 2^{3m} \cdot 3^{2m}$$

   c. What value of m could be substituted into the identity in part (b) to find the answer to part (a)?

3.

a. Jill writes $2^3 \cdot 4^3 = 8^6$ and the teacher marked it wrong. Explain Jill's error.

b. Find n so that the number sentence below is true:
$2^3 \cdot 4^3 = 2^3 \cdot 2^n = 2^9$

c. Use the definition of exponential notation to demonstrate why $2^3 \cdot 4^3 = 2^9$ is true.

d. You write $7^5 \cdot 7^{-9} = 7^{-4}$. Keisha challenges you, "Prove it!" Show directly why your answer is correct without referencing the laws of exponents for integers; in other words, $x^a \cdot x^b = x^{a+b}$ for positive numbers x and integers a and b.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a–d** 8.EE.A.1	Student answered 0–1 parts of (a)–(d) correctly. Student was able to complete the table for at least values of 0–5 for part (b). Student was unable to respond to questions or left items blank.	Student answered 2–3 parts of (a)–(d) correctly. Student was able to complete the table in part (b) correctly for 5 or more entries, including at least one value on each side of the value given for year 1. Student provided a limited expression of reasoning in parts (c) and (d).	Student answered 3–4 parts of (a)–(d) correctly. Student provided correct answers with some reasoning for making calculations OR Student had a few miscalculations but provided substantial reasoning with proper use of grade-level vocabulary.	Student answered all parts of (a)–(d) correctly. Student provided solid reasoning for making calculations with proper use of grade-level vocabulary.
	e–g 8.EE.A.1	Student answered 0–1 parts of (e)–(g) correctly. Student was unable to relate the pattern in the problem to exponential growth.	Student answered 1–2 parts of (e)–(g) correctly. Student was able to relate the pattern in the problem to exponential growth by writing an equation. Student justifications were incomplete.	Student answered 2–3 parts of (e)–(g) correctly. Equation given was correct, and student was able to answer questions, but justifications were incomplete. OR The equation given related the pattern to exponential growth, but was incomplete or contained a minor error, AND student was able to answer questions using solid reasoning based on the information provided.	Student answered all parts of (e)–(g) correctly. Student justified answers and made accurate conclusions based on the information provided in the problem. Student was able to explain limitations of equation when looking ahead in time and back in time.

2	a 8.EE.A.1	Student answered incorrectly. No evidence of use of properties of exponents.	Student answered incorrectly. Properties of exponents were used incorrectly.	Student answered correctly. Some evidence of use of properties of exponents is shown in calculations.	Student answered correctly. Student provided substantial evidence of the use of properties of exponents to simplify the expression to distinct primes.
	b–c 8.EE.A.1	Student answered parts (b)–(c) incorrectly. No evidence of use of properties of exponents.	Student answered parts (b)–(c) incorrectly. Properties of exponents were used incorrectly.	Student answered part (b) and/or part (c) correctly. Some evidence of use of properties of exponents is shown in calculations.	Student answered both parts (b) and (c) correctly. Student provided substantial evidence of the use of properties of exponents to prove the identity.
3	a 8.EE.A.1	Student stated that Jill's response was correct. OR Student was unable to identify the mistake and provided no additional information.	Student stated that Jill's answer was incorrect. Student was unable to identify the mistake of multiplying unlike bases. Student may have used what he or she knows about exponential notation to multiply numbers to show the answer was incorrect.	Student identified Jill's error as "multiplied unlike bases."	Student identified Jill's error as "multiplied unlike bases." Student provided a thorough explanation as to how unlike bases can be rewritten so that properties of exponents can be used properly.
	b 8.EE.A.1	Student was unable to identify the correct value for n.	Student correctly answered $n = 6$. No explanation was provided as to why the answer is correct.	Student correctly answered $n = 6$. Student stated that $4^3 = 2^6$ with little or no explanation or work shown.	Student correctly answered $n = 6$. Student *clearly showed* that 4^3 is equivalent to 2^6.
	c 8.EE.A.1	Student used the definition of exponential notation to rewrite 4^3 as $4 \times 4 \times 4$. Student was unable to complete the problem.	Student multiplied 4^3 to get 64 and was able to rewrite it as a number with a base of 2 but had the wrong exponent.	Student correctly rewrote 4^3 as 2^6, and then used the first property of exponents to show that the answer was correct.	Student correctly rewrote 4^3 as 2^6. Student used definition of exponential notation to rewrite each number as repeated multiplication. Student clearly showed how/why the exponents are added to simplify such expressions.

	d **8.EE.A.1**	Student may have been able to rewrite 7^{-9} as a fraction but was unable to operate with fractions.	Student was unable to show why part (d) was correct but may have used a property of exponents to state that the given answer was correct.	Student answered part (d) but misused or left out definitions in explanations and proofs.	Student answered part (d) correctly and used definitions and properties to thoroughly explain and prove the answer. Answer showed strong evidence that student understands exponential notation and can use the properties of exponents proficiently.

Name ____ Date ____________________

1. The number of users of social media has increased significantly since the year 2001. In fact, the approximate number of users has tripled each year. It was reported that in 2005 there were 3 million users of social media.

   a. Assuming that the number of users continues to triple each year, for the next three years, determine the number of users in 2006, 2007, and 2008.

   2006 – 9 MILLION
   2007 – 27 MILLION
   2008 - 81 MILLION

   b. Assume the trend in the numbers of users tripling each year was true for all years from 2001 to 2009. Complete the table below using 2005 as year 1 with 3 million as the number of users that year.

Year	-3	-2	-1	0	1	2	3	4	5
# of users in millions	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243

   c. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 2, 3, 4, and 5?

   I MULTIPLIED THE PRECEDING YEAR'S NUMBER OF USERS BY 3.

   d. Given only the number of users in 2005 and the assumption that the number of users triples each year, how did you determine the number of users for years 0, −1, −2, and −3?

   I DIVIDED THE NEXT YEAR'S NUMBER OF USERS BY 3.

e. Write an equation to represent the number of users in millions, N, for year, t, $t \geq -3$.

$N = 3^t$

f. Using the context of the problem, explain whether or not the formula, $N = 3^t$ would work for finding the number of users in millions in year t, for all $t \leq 0$.

WE ONLY KNOW THAT THE NUMBER OF USERS HAS TRIPLED EACH YEAR IN THE TIME FRAME OF 2001 TO 2009. FOR THAT REASON, WE CANNOT RELY ON THE FORMULA, $N=3^t$, TO WORK FOR ALL $t \leq 0$, JUST TO $t=-3$, WHICH IS THE YEAR 2001.

g. Assume the total number of users continues to triple each year after 2009. Determine the number of users in 2012. Given that the world population at the end of 2011 was approximately 7 billion, is this assumption reasonable? Explain your reasoning.

2012 IS $t=8$, SO WHEN $t=8$ IN $N=3^t$, $N=6{,}561{,}000{,}000$. THE NUMBER OF USERS IN 2012, 6,561,000,000 DOES NOT EXCEED THE WORLD POPULATION OF 7 BILLION, THEREFORE IT IS POSSIBLE TO HAVE THAT NUMBER OF USERS. BUT 6,561,000,000 IS APPROXIMATELY 94% OF THE WORLD'S POPULATION. THE NUMBER OF USERS IS LIKELY LESS THAN THAT DUE TO POVERTY, ILLNESS, INFANCY, ETC. THE ASSUMPTION IS POSSIBLE, BUT NOT REASONABLE.

2. Let m be a whole number.

   a. Use the properties of exponents to write an equivalent expression that is a product of unique primes, each raised to an integer power.

$$\frac{6^{21} \cdot 10^7}{30^7} = \frac{(3\cdot 2)^{21}\cdot 10^7}{(3\cdot 10)^7} = \frac{3^{21}\cdot 2^{21}\cdot 10^7}{3^7\cdot 10^7}$$
$$= 3^{21-7}\cdot 2^{21}\cdot 10^{7-7}$$
$$= 3^{14}\cdot 2^{21}\cdot 10^0$$
$$= 3^{14}\cdot 2^{21}$$

   b. Use the properties of exponents to prove the following identity:

$$\frac{6^{3m} \cdot 10^m}{30^m} = 2^{3m} \cdot 3^{2m}$$

$$\frac{6^{3m}\cdot 10^m}{30^m} = \frac{(3\cdot 2)^{3m}\cdot 10^m}{(3\cdot 10)^m}$$
$$= \frac{3^{3m}\cdot 2^{3m}\cdot 10^m}{3^m\cdot 10^m}$$
$$= 3^{3m-m}\cdot 2^{3m}\cdot 10^{m-m} = 3^{2m}\cdot 2^{3m} = 2^{3m}\cdot 3^{2m}$$

   c. What value of m could be substituted into the identity in part (b) to find the answer to part (a)?

$$2^{3m}\cdot 3^{2m} = 2^{21}\cdot 3^{14}$$

$3m = 21$ $\quad$ $2m = 14$

$m = 7$ $\quad$ $m = 7$

THEREFORE, $m = 7$.

3.

a. Jill writes $2^3 \cdot 4^3 = 8^6$ and the teacher marked it wrong. Explain Jill's error.

JILL MULTIPLIED THE BASES, 2 AND 4, AND ADDED THE EXPONENTS. YOU CAN ONLY ADD THE EXPONENTS WHEN THE BASES BEING MULTIPLIED ARE THE SAME.

b. Find n so that the number sentence below is true:
$2^3 \cdot 4^3 = 2^3 \cdot 2^n = 2^9$

$4^3 = 4 \cdot 4 \cdot 4$
$= (2 \cdot 2)(2 \cdot 2)(2 \cdot 2)$
$= 2^6$

THEREFORE:
$2^3 \cdot 4^3 = 2^3 \cdot 2^6 = 2^9$
SO $n = 6$

c. Use the definition of exponential notation to demonstrate why $2^3 \cdot 4^3 = 2^9$ is true.

$4^3 = 2^6$, SO $2^3 \cdot 4^3 = 2^9$ IS EQUIVALENT TO $2^3 \cdot 2^6 = 2^9$.

BY DEFINITION OF EXPONENTIAL NOTATION:

$2^3 \cdot 2^6 = \underbrace{(2 \times \cdots \times 2)}_{3 \text{ times}} \times \underbrace{(2 \times \cdots \times 2)}_{6 \text{ times}} = \underbrace{(2 \times \cdots \times 2)}_{3+6 \text{ times}} = 2^{3+6} = 2^9$

d. You write $7^5 \cdot 7^{-9} = 7^{-4}$. Keisha challenges you, "Prove it!" Show directly why your answer is correct without referencing the laws of exponents for integers; in other words, $x^a \cdot x^b = x^{a+b}$ for positive numbers x and integers a and b.

$7^5 \cdot 7^{-9} = 7^5 \cdot \frac{1}{7^9}$ BY DEFINITION

$= \frac{7^5}{7^9}$ BY PRODUCT FORMULA

$= \frac{7^5}{7^5 \cdot 7^4}$ BY $x^m \cdot x^n = x^{m+n}$ for $x > 0$, $m, n \geq 0$

$= \frac{1}{7^4}$ BY EQUIVALENT FRACTIONS

$= 7^{-4}$ BY DEFINITION.