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GRADE 3 • MODULE 1

Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

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Grade 3 • Module 1

Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

OVERVIEW

This 25-day module begins the year by building on students' fluency with addition and their knowledge of arrays. In Topic A, students initially use repeated addition to find the total from a number of equal groups (**2.OA.4**). As students notice patterns, they let go of longer addition sentences in favor of more efficient multiplication facts (**3.OA.1**). Lessons in Topic A move students' Grade 2 work with arrays and repeated addition a step further by developing skip-counting rows as a strategy for multiplication. Arrays become a cornerstone of the module. Students use the language of multiplication as they understand what factors are and differentiate between the size of groups and the number of groups within a given context. In this module, the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

The study of factors links Topics A and B; Topic B extends the study to division. Students understand division as an unknown factor problem and relate the meaning of unknown factors to either the number or the size of groups (**3.OA.2, 3.OA.6**). By the end of Topic B, students are aware of a fundamental connection between multiplication and division that lays the foundation for the rest of the module.

In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication and practice related facts of 2 and 3. They become fluent enough with arithmetic patterns to “add” or “subtract” groups from known products to solve more complex multiplication problems (**3.OA.1**). They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor (**3.OA.3**). This culminates in students using arrays to model the distributive property as they decompose units to multiply (**3.OA.5**).

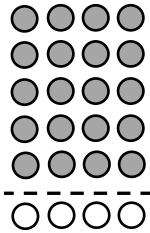
In Topic D, students model, write, and solve partitive and measurement division problems with 2 and 3 (**3.OA.2**). Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division. Modeling advances as students use tape diagrams to represent multiplication and division. A final lesson in this topic solidifies a growing understanding of the relationship between operations (**3.OA.7**).

Topic E shifts students from simple understanding to analyzing the relationship between multiplication and division. Practice of both operations is combined—this time using units of 4—and a lesson is explicitly dedicated to modeling the connection between them (**3.OA.7**).

Skip-counting, the distributive property, arrays, number bonds, and tape diagrams are tools for both

The Distributive Property

$6 \times 4 = \underline{\quad}$



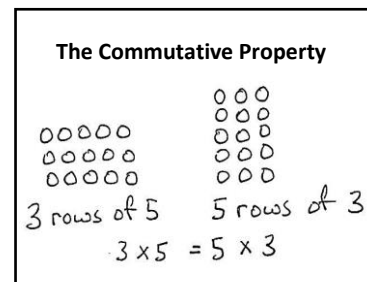
$(5 \times 4) = 20$

$(1 \times 4) = 4$

$(6 \times 4) = (5 \times 4) + (1 \times 4)$
 $= 20 + 4$

operations (**3.OA.1, 3.OA.2**). A final lesson invites students to explore their work with arrays and related facts through the lens of the commutative property as it relates to multiplication (**3.OA.5**).

Topic F introduces the factors 5 and 10, familiar from skip-counting in Grade 2. Students apply the multiplication and division strategies they have used to mixed practice with all of the factors included in Module 1 (**3.OA.1, 3.OA.2, 3.OA.3**). Students model relationships between factors, analyzing the arithmetic patterns that emerge to compose and decompose numbers, as they further explore the relationship between multiplication and division (**3.OA.3, 3.OA.5, 3.OA.7**).



In the final lesson of the module, students apply the tools, representations, and concepts they have learned to problem solving with multi-step word problems using all four operations (**3.OA.3, 3.OA.8**). They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.

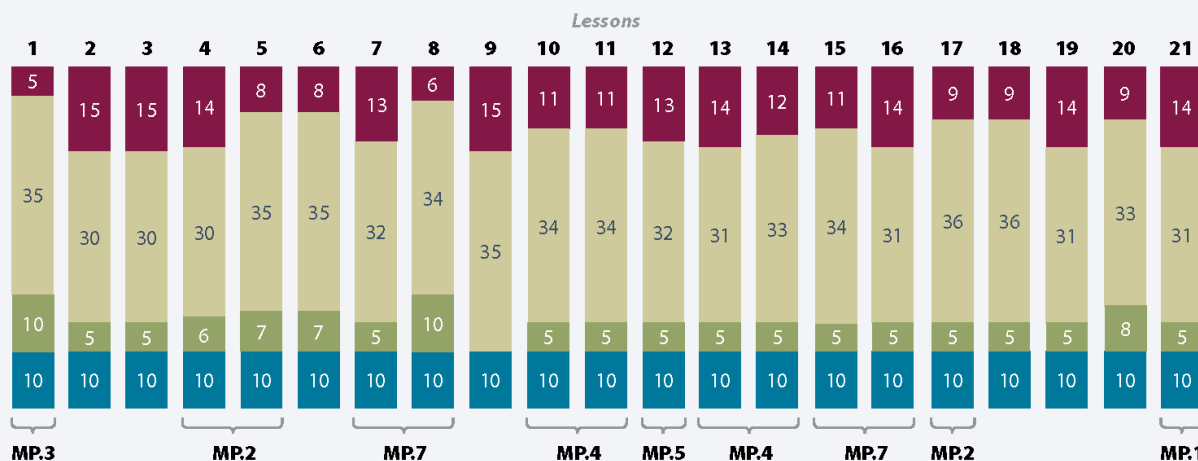
The Mid-Module Assessment follows Topic C. The End-of-Module Assessment follows Topic F.



Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- Fluency Practice
- Concept Development
- Application Problems
- Student Debrief



Focus Grade Level Standards

Represent and solve problems involving multiplication and division.¹

- 3.OA.1** Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*
- 3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*
- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*

Understand properties of multiplication and the relationship between multiplication and division.²

- 3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)³*
- 3.OA.6** Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.⁴

- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in

¹ Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

² Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

³ The associative property is addressed in Module 3.

⁴ Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

arithmetic.⁵

- 3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

Foundational Standards

- 2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
- 2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
- 2.NBT.2** Count within 1000; skip-count by 5s, 10s, and 100s.

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students model multiplication and division using the array model. They solve two-step mixed word problems and assess the reasonableness of their solutions.
- MP.2** **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships as they explore the properties of multiplication and division and the relationship between them. Students decontextualize when representing equal group situations as multiplication and when they represent division as partitioning objects into equal shares or as unknown factor problems. Students contextualize when they consider the value of units and understand the meaning of the quantities as they compute.
- MP.3** **Construct viable arguments and critique the reasoning of others.** Students represent and solve multiplication and division problems using arrays and equations. As they compare methods, they construct arguments and critique the reasoning of others. This practice is particularly exemplified in daily Application Problems and in specific lessons dedicated to problem solving in which students solve and reason with others about their work.
- MP.4** **Model with mathematics.** Students represent equal groups using arrays and equations to multiply, divide, add, and subtract.
- MP.7** **Look for and make use of structure.** Students notice structure when they represent quantities by using drawings and equations to represent the commutative and distributive properties. The relationship between multiplication and division also highlights structure for students as they determine the unknown whole number in a multiplication or division equation.

⁵ In this module, problem solving is limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed in Module 3.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives	Days
3.OA.1 3.OA.3	A Multiplication and the Meaning of the Factors Lesson 1: Understand <i>equal groups of</i> as multiplication. Lesson 2: Relate multiplication to the array model. Lesson 3: Interpret the meaning of factors—the size of the group or the number of groups.	3
3.OA.2 3.OA.6 3.OA.3 3.OA.4	B Division as an Unknown Factor Problem Lesson 4: Understand the meaning of the unknown as the size of the group in division. Lesson 5: Understand the meaning of the unknown as the number of groups in division. Lesson 6: Interpret the unknown in division using the array model.	3
3.OA.1 3.OA.5 3.OA.3 3.OA.4	C Multiplication Using Units of 2 and 3 Lessons 7–8: Demonstrate the commutativity of multiplication, and practice related facts by skip-counting objects in array models. Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models. Lesson 10: Model the distributive property with arrays to decompose units as a strategy to multiply.	4
	Mid-Module Assessment: Topics A–C (assessment ½ day, return ½ day, remediation or further applications 1 day)	2
3.OA.2 3.OA.4 3.OA.6 3.OA.7 3.OA.3 3.OA.8	D Division Using Units of 2 and 3 Lesson 11: Model division as the unknown factor in multiplication using arrays and tape diagrams. Lesson 12: Interpret the quotient as the number of groups or the number of objects in each group using units of 2. Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3.	3



Standards	Topics and Objectives	Days
3.OA.5 3.OA.7 3.OA.1 3.OA.2 3.OA.3 3.OA.4 3.OA.6	E Multiplication and Division Using Units of 4 Lesson 14: Skip-count objects in models to build fluency with multiplication facts using units of 4. Lesson 15: Relate arrays to tape diagrams to model the commutative property of multiplication. Lesson 16: Use the distributive property as a strategy to find related multiplication facts. Lesson 17: Model the relationship between multiplication and division.	4
3.OA.3 3.OA.5 3.OA.7 3.OA.8 3.OA.1 3.OA.2 3.OA.4 3.OA.6	F Distributive Property and Problem Solving Using Units of 2–5 and 10 Lessons 18–19: Apply the distributive property to decompose units. Lesson 20: Solve two-step word problems involving multiplication and division, and assess the reasonableness of answers. Lesson 21: Solve two-step word problems involving all four operations, and assess the reasonableness of answers.	4
	End-of-Module Assessment: Topics A–F (assessment ½ day, return ½ day, remediation or further application 1 day)	2
Total Number of Instructional Days		25

Terminology

New or Recently Introduced Terms

- Array⁶ (arrangement of objects in rows and columns)
- Commutative property/commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Distribute (with reference to the distributive property, e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$ the 3 is the multiplier for each part of the decomposition)
- Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)

⁶ Originally introduced in Grade 2, Module 6 but treated as new vocabulary in this module.

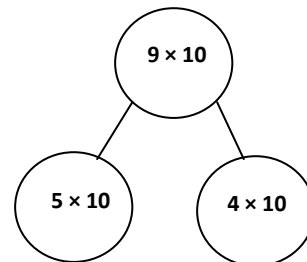
- Factors (numbers that are multiplied to obtain a product)
- Multiplication/multiply (an operation showing how many times a number is added to itself, e.g., $5 \times 3 = 15$)
- Number of groups (factor in a multiplication problem that refers to the total equal groups)
- Parentheses (symbols () used around an expression or numbers within an equation)
- Quotient (the answer when one number is divided by another)
- Rotate (turn, used with reference to turning arrays 90 degrees)
- Row/column⁷ (in reference to rectangular arrays)
- Size of groups (factor in a multiplication problem that refers to how many in a group)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the missing factor or quantity in multiplication or division)

Familiar Terms and Symbols⁸

- Add 1 unit, subtract 1 unit (add or subtract a single unit of two, ten, etc.)
- Expression (see expanded description in box above)
- Number bond (illustrates part–part–whole relationship, shown at right)
- Ones, twos, threes, etc. (units of one, two, or three)
- Repeated addition (adding equal groups together, e.g., $2 + 2 + 2 + 2$)
- Tape diagram (a method for modeling problems)
- Value (how much)

Suggested Tools and Representations

- 18 counters per student
- Tape diagram (a method for modeling problems)
- Number bond (shown at right)
- Array (arrangement of objects in rows and columns)



NOTES ON EXPRESSION, EQUATION, AND NUMBER SENTENCE:

Please note the descriptions for the following terms, which are frequently misused.

- **Expression:** A number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number (e.g., $3 + 4$, 8×3 , $15 \div 3$ as distinct from an equation or number sentence).
- **Equation:** A statement that two expressions are equal (e.g., $3 \times \underline{\quad} = 12$, $5 \times b = 20$, $3 + 2 = 5$).
- **Number sentence** (also addition, subtraction, multiplication, or division sentence): An equation or inequality for which both expressions are numerical and can be evaluated to a single number (e.g., $4 + 3 = 6 + 1$, $2 = 2$, $21 > 7 \times 2$, $5 \div 5 = 1$). Number sentences are either true or false (e.g., $4 + 4 < 6 \times 2$ and $21 \div 7 = 4$) and contain no unknowns.

⁷ Originally introduced in Grade 2, Module 6 but treated as new vocabulary in this module.

⁸ These are terms and symbols students have used or seen previously.

Suggested Methods of Instructional Delivery

Directions for Administration of Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is celebrated.

One Sprint has two parts with closely related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

With practice, the following routine takes about 9 minutes.

Sprint A

Pass Sprint A out quickly, face down on student desks with instructions to not look at the problems until the signal is given. (Some Sprints include words. If necessary, prior to starting the Sprint, quickly review the words so that reading difficulty does not slow students down.)

T: You will have 60 seconds to do as many problems as you can. I do not expect you to finish all of them. Just do as many as you can, your personal best. (If some students are likely to finish before time is up, assign a number to count by on the back.)

T: Take your mark! Get set! THINK!

Students immediately turn papers over and work furiously to finish as many problems as they can in 60 seconds. Time precisely.

T: Stop! Circle the last problem you did. I will read just the answers. If you got it right, call out "Yes!" If you made a mistake, circle it. Ready?

T: (Energetically, rapid-fire call the first answer.)

S: Yes!

T: (Energetically, rapid-fire call the second answer.)

S: Yes!

Repeat to the end of Sprint A or until no student has a correct answer. If needed, read the count-by answers in the same way you read Sprint answers. Each number counted-by on the back is considered a correct answer.

T: Fantastic! Now, write the number you got correct at the top of your page. This is your personal goal for Sprint B.

T: How many of you got one right? (All hands should go up.)

T: Keep your hand up until I say the number that is one more than the number you got correct. So, if you got 14 correct, when I say 15, your hand goes down. Ready?

T: (Continue quickly.) How many got two correct? Three? Four? Five? (Continue until all hands are down.)

If the class needs more practice with Sprint A, continue with the optional routine presented below.

T: I'll give you one minute to do more problems on this half of the Sprint. If you finish, stand behind your chair.

As students work, the student who scored highest on Sprint A might pass out Sprint B.

T: Stop! I will read just the answers. If you got it right, call out "Yes!" If you made a mistake, circle it. Ready? (Read the answers to the first half again as students stand.)

Movement

To keep the energy and fun going, always do a stretch or a movement game in between Sprints A and B. For example, the class might do jumping jacks while skip-counting by 5 for about 1 minute. Feeling invigorated, students take their seats for Sprint B, ready to make every effort to complete more problems this time.

Sprint B

Pass Sprint B out quickly, face down on student desks with instructions to not look at the problems until the signal is given. (Repeat the procedure for Sprint A up through the show of hands for how many right.)

T: Stand up if you got more correct on the second Sprint than on the first.

S: (Stand.)

T: Keep standing until I say the number that tells how many more you got right on Sprint B. If you got three more right on Sprint B than you did on Sprint A, when I say *three*, you sit down. Ready? (Call out numbers starting with one. Students sit as the number by which they improved is called. Celebrate the students who improved most with a cheer.)

T: Well done! Now, take a moment to go back and correct your mistakes. Think about what patterns you noticed in today's Sprint.

T: How did the patterns help you get better at solving the problems?

T: Rally Robin your thinking with your partner for 1 minute. Go!

Rally Robin is a style of sharing in which partners trade information back and forth, one statement at a time per person, for about 1 minute. This is an especially valuable part of the routine for students who benefit from their friends' support to identify patterns and try new strategies.

Students may take Sprints home.

RDW or Read, Draw, Write (an Equation and a Statement)

Mathematicians and teachers suggest a simple process applicable to all grades:

- 1) Read.
- 2) Draw and Label.
- 3) Write an equation.
- 4) Write a word sentence (statement).

The more students participate in reasoning through problems with a systematic approach, the more they internalize those behaviors and thought processes.

- What do I see?
- Can I draw something?
- What conclusions can I make from my drawing?

Modeling with Interactive Questioning	Guided Practice	Independent Practice
The teacher models the whole process with interactive questioning, some choral response, and talk such as “What did Monique say, everyone?” After completing the problem, students might reflect with a partner on the steps they used to solve the problem. “Students, think back on what we did to solve this problem. What did we do first?” Students might then be given the same or a similar problem to solve for homework.	Each student has a copy of the question. Though guided by the teacher, they work independently at times and then come together again. Timing is important. Students might hear, “You have 2 minutes to do your drawing.” Or, “Put your pencils down. Time to work together again.” The Debrief might include selecting different student work to share.	The students are given a problem to solve and possibly a designated amount of time to solve it. The teacher circulates, supports, and is thinking about which student work to show to support the mathematical objectives of the lesson. When sharing student work, students are encouraged to think about the work with questions such as, “What do you see Jeremy did?” “What is the same about Jeremy’s work and Sara’s work?” “How did Jeremy show $\frac{7}{7}$ of the students?” “How did Sara show $\frac{3}{7}$ of the students?”

Personal White Boards

Materials Needed for Personal White Boards

- 1 heavy duty clear sheet protector
- 1 piece of stiff red tag board $11" \times 8 \frac{1}{4}"$
- 1 piece of stiff white tag board $11" \times 8 \frac{1}{4}"$
- 1 $3" \times 3"$ piece of dark synthetic cloth for an eraser (e.g., felt)
- 1 low odor blue dry erase marker, fine point

Directions for Creating Personal White Boards

Cut your white and red tag to specifications. Slide into the sheet protector. Store your eraser on the red side. Store markers in a separate container to avoid stretching the sheet protector.

Frequently Asked Questions About Personal White Boards

Why is one side red and one white?

The white side of the board is the “paper.” Students generally write on it, and if working individually, turn the board over to signal to the teacher they have completed their work. The teacher then says, “Show me your boards,” when most of the class is ready.

What are some of the benefits of a personal white board?

- The teacher can respond quickly to a gap in student understandings and skills. “Let’s do some of these on our personal white boards until we have more mastery.”
- Students can erase quickly so that they do not have to suffer the evidence of their mistake.
- They are motivating. Students love both the drill and thrill capability and the chance to do story problems with an engaging medium.
- Checking work gives the teacher instant feedback about student understanding.

What is the benefit of this personal white board over a commercially purchased dry erase board?

- It is much less expensive.
- Templates such as place value charts, number bond mats, hundreds boards, and number lines can be stored between the two pieces of tag board for easy access and reuse.
- Worksheets, story problems, and other problem sets can be done without marking the paper so that students can work on the problems independently at another time.
- Strips with story problems, number lines, and arrays can be inserted and still have a full piece of paper on which to write.
- The red versus white side distinction clarifies your expectations. When working collaboratively, there is no need to use the red. When working independently, the students know how to keep their work private.
- The tag board can be removed so that student work can be projected on an overhead.

Scaffolds⁹

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	3.OA.1 3.OA.2 3.OA.5 3.OA.6
End-of-Module Assessment Task	After Topic F	Constructed response with rubric	3.OA.1 3.OA.2 3.OA.3 3.OA.4 3.OA.5 3.OA.6 3.OA.7 3.OA.8

⁹ Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website, www.p12.nysed.gov/specialed/aim, for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.