

Lesson 18: Analyzing Decisions and Strategies Using Probability

Classwork

Exercise 1

Suppose that someone is offering to sell you raffle tickets. There are blue, green, yellow, and red tickets available. Each ticket costs the same to purchase regardless of color. The person selling the tickets tells you that 369 blue tickets, 488 green tickets, 523 yellow tickets, and 331 red tickets have been sold. At the drawing, one ticket of each color will be drawn, and four identical prizes will be awarded. Which color ticket would you buy? Explain your answer.

Exercise 2

Suppose that you are taking part in a TV game show. The presenter has a set of 60 cards, 10 of which are red and the rest are blue. The presenter randomly splits the cards into two piles and places one on your left and one on your right. The presenter tells you that there are 32 blue cards in the pile on your right. You look at the pile of cards on your left and estimate that it contains 24 cards. You will be given a chance to pick a card at random, and you know that if you pick a red card you will win \$5,000.00. If you pick a blue card, you will get nothing. The presenter gives you the choice of picking a card at random from the pile on the left, from the pile on the right, or from the entire set of cards. Which should you choose? Explain your answer.

Exercise 3

The American Lyme Disease Foundation states that a commonly used test called the ELISA test will be positive for virtually all patients who have the disease but that the test is also positive for around 6% of those who do not have the disease. (<http://www.aldf.com/faq.shtml#Testing>) For the purposes of this question, assume that the ELISA test is positive for all patients who have the disease and for 6% of those who do not have the disease. Suppose the test is performed on a randomly selected resident of Connecticut where, according to the Centers for Disease Control and Prevention, 46 out of every 100,000 people have Lyme disease.

(http://www.cdc.gov/lyme/stats/chartstables/reportedcases_statelocality.html)

- a. Complete the hypothetical 100,000-person two-way frequency table below for 100,000 Connecticut residents.

	Test is Positive	Test is Negative	Total
Has the disease			
Does not have the disease			
Total			100,000

- b. If a randomly selected person from Connecticut is tested for the disease using the ELISA test and the test is positive, what is the probability that the person has the disease? (Round your answer to the nearest thousandth.)
- c. Comment on your answer to part (b). What should the medical response be if a person is tested using the ELISA test and the test is positive?

Example 1

You are playing a game that uses a deck of cards consisting of 10 green, 10 blue, 10 purple, and 10 red cards. You will select four cards at random, and you want all four cards to be the same color. You are given two alternatives. You can randomly select the four cards one at a time, with each card being returned to the deck and the deck being shuffled before you pick the next card. Alternatively, you can randomly select four cards without the cards being returned to the deck. Which should you choose? Explain your answer.

Exercise 4

You are at a stall at a fair where you have to throw a ball at a target. There are two versions of the game. In the first version, you are given three attempts, and you estimate that your probability of success on any given throw is 0.1. In the second version, you are given five attempts, but the target is smaller, and you estimate that your probability of success on any given throw is 0.05. The prizes for the two versions of the game are the same, and you are willing to assume that the outcomes of your throws are independent. Which version of the game should you choose? (Hint: In the first version of the game, the probability that you do not get the prize is the probability that you fail on all three attempts.)

Lesson Summary

If a number of strategies are available and the possible outcomes are success and failure, the best strategy is the one that has the highest probability of success.

Problem Set

1. Jonathan is getting dressed in the dark. He has three drawers of socks. The top drawer contains 5 blue and 5 red socks, the middle drawer contains 6 blue and 4 red socks, and the bottom drawer contains 3 blue and 2 red socks. Jonathan will open one drawer and will select two socks at random.
 - a. Which drawer should he choose in order to make it most likely that he will select 2 red socks?
 - b. Which drawer should he choose in order to make it most likely that he will select 2 blue socks?
 - c. Which drawer should he choose in order to make it most likely that he will select a matching pair?
2. Commuters in London have the problem that buses are often already full and, therefore, cannot take any further passengers. Sarah is heading home from work. She has the choice of going to Bus Stop A, where there are three buses per hour and 30% of the buses are full, or Bus Stop B, where there are four buses per hour and 40% of the buses are full. Which stop should she choose in order to maximize the probability that she will be able to get on a bus within the next hour? (Hint: Calculate the probability, for each bus stop, that she will fail to get on a bus within the next hour. You may assume that the buses are full, or not, independently of each other.)
3. An insurance salesman has been told by his company that about 20% of the people in a city are likely to buy life insurance. Of those who buy life insurance, around 30% own their homes, and of those who do not buy life insurance, around 10% own their homes. In the questions that follow, assume that these estimates are correct.
 - a. If a homeowner is selected at random, what is the probability that the person will buy life insurance? (Hint: Use a hypothetical 1,000-person two-way frequency table.)
 - b. If a person is selected at random from those who do not own their homes, what is the probability that the person will buy life insurance?
 - c. Is the insurance salesman better off trying to sell life insurance to homeowners or to people who do not own their homes?
4. You are playing a game. You are given the choice of rolling a fair six-sided number cube (with faces labeled 1–6) three times or selecting three cards at random from a deck that consists of
 - 4 cards labeled 1
 - 4 cards labeled 2
 - 4 cards labeled 3
 - 4 cards labeled 4
 - 4 cards labeled 5
 - 4 cards labeled 6

If you decide to select from the deck of cards, then you will not replace the cards in the deck between your selections. You will win the game if you get a triple (that is, rolling the same number three times or selecting three cards with the same number). Which of the two alternatives, the number cube or the cards, will make it more likely that you will get a triple? Explain your answer.

5. There are two routes Jasmine can take to work. Route A has five stoplights. The probability distribution of how many lights at which she will need to stop is below. The average amount of time spent at each stoplight on Route A is 30 seconds.

Number of Red Lights	0	1	2	3	4	5
Probability	0.04	0.28	0.37	0.14	0.11	0.06

Route B has three stoplights. The probability distribution of Route B is below. The average wait time for these lights is 45 seconds.

Number of Red Lights	0	1	2	3
Probability	0.09	0.31	0.40	0.20

- In terms of stopping at the least number of stoplights, which route may be the best for Jasmine to take?
 - In terms of least time spent at stoplights, which route may be the best for Jasmine to take?
6. A manufacturing plant has been short-handed lately, and one of its plant managers recently gathered some data about shift length and frequency of work-related accidents in the past month (accidents can range from forgetting safety equipment to breaking a nail to other, more serious injuries). Below is the table displaying his findings.

	Number of shifts with 0 accidents	Number of shifts with at least one accident	Total
$0 < x < 8$ hours	815	12	827
$8 \leq x \leq 10$ hours	53	27	80
$x > 10$ hours	52	41	93
Total	920	80	1,000

- What is the probability that a person had an accident?
- What happens to the accident likelihood as the number of hours increases?
- What are some options the plant could pursue in order to try to cut down or eliminate accidents?