

Lesson 17: Fair Games

Classwork

Example 1: What Is a Fair Game?

An instant lottery game card consists of six disks labeled A, B, C, D, E, F. The game is played by purchasing a game card and scratching off two disks. Each of five of the disks hides \$1.00, and one of the disks hides \$10.00. The total of the amounts on the two disks that are scratched off is paid to the person who purchased the card.

Exercises 1–5

1. What are the possible total amounts of money you could win if you scratch off two disks?
2. If you pick two disks at random:
 - a. How likely is it that you win \$2.00?
 - b. How likely is it that you win \$11.00?

3. Based on Exercise 3, how much should you expect to win on average per game if you played this game a large number of times?

4. To play the game, you must purchase a game card. The price of the card is set so that the game is fair. What do you think is meant by a fair game in the context of playing this instant lottery game?

5. How much should you be willing to pay for a game card if the game is to be a fair one? Explain.

Example 2: Deciding Between Two Alternatives

You have a chore to do around the house for which your mom plans to pay you \$10.00. When you are done, your mom, being a mathematics teacher, gives you the opportunity to change the amount that you are paid by playing a game. She puts three \$2.00 bills in a bag along with two \$5.00 bills and one \$20.00 bill. She says that you can take the \$10.00 she offered originally or you can play the game by reaching into the bag and selecting two bills without looking. You get to keep these two bills as your payment

Exercises 6–7

6. Do you think you should take your mom’s original payment of \$10.00 or play the “bag” game? In other words, is this game a fair alternative to getting paid \$10.00? Use a probability distribution to help answer this question.
7. Alter the contents of the bag in Example 2 to create a game that would be a fair alternative to getting paid \$10.00. You must keep six “bill” in the bag, but you can choose to include bill-sized pieces of paper that are marked as \$0.00 to represent a \$0.0 bill.

Example 3: Is an Additional Year of Warranty Worth Purchasing?

Suppose you are planning to buy a computer. The computer comes with a one-year warranty, but you can purchase a warranty for an additional year for \$24.95. Your research indicates that in the second year, there is a 1 in 20 chance of incurring a major repair that costs \$180.00 and a probability of 0.15 of a minor repair that costs \$65.00.

Exercises 8–9

8. Is it worth purchasing the additional year warranty? Why or why not?

9. If the cost of the additional year warranty is too high, what would be a fair price to charge?

Example 4: Spinning a Pentagon

Your math club is sponsoring a game tournament to raise money for the club. The game is to spin a fair pentagon spinner twice and add the two outcomes. The faces of the spinner are numbered 1, 2, 3, 4, and 5. If the sum is odd, you win; if the sum is even, the club wins.

Exercises 10–11

10. The math club is trying to decide what to charge to play the game and what the winning payoff should be per play to make it a fair game. Give an example.

11. What should the math club charge per play to make \$0.25 on average for each game played? Justify your answer.

Lesson Summary

- The concept of fairness in statistics requires that one outcome is not favored over another.
- In a game that involves a fee to play, a game is fair if the amount paid for one play of the game is the same as the expected winnings in one play.

Problem Set

1. A game is played by drawing a single card from a regular deck of playing cards. If you get a black card, you win nothing. If you get a diamond, you win \$5.00. If you get a heart, you win \$10.00. How much would you be willing to pay if the game is to be fair? Explain.
2. Suppose that for the game described in Problem 1, you win a bonus for drawing the queen of hearts. How would that change the amount you are willing to pay for the game? Explain.
3. You are trying to decide between playing two different carnival games and want to only play games that are fair. One game involves throwing a dart at a balloon. It costs \$10.00 to play, and if you break the balloon with one throw, you win \$75.00. If you do not break the balloon, you win nothing. You estimate that you have about a 15% chance of breaking the balloon.

The other game is a ring toss. For \$5.00 you get to toss three rings and try to get them around the neck of a bottle. If you get one ring around a bottle, you win \$3.00. For two rings around the bottle, you win \$15.00. For three rings, you win \$75.00. If no rings land around the neck of the bottle, you win nothing. You estimate that you have about a 15% chance of tossing a ring and it landing around the neck of the bottle. Each toss of the ring is independent. Which game will you play? Explain.
4. Invent a fair game that involves three fair number cubes. State how the game is played and how the game is won. Explain how you know the game is fair.
5. Invent a game that is not fair that involves three fair number cubes. State how the game is played and how the game is won. Explain how you know the game is not fair.