Lesson 15: Using Expected Values to Compare Strategies

Student Outcomes

- Students calculate expected values.
- Students make rational decisions based on calculated expected values.

Lesson Notes

In previous lessons, students analyzed and interpreted simple games of chance by calculating expected payoff. Further, they used expected payoff to compare strategies for such games. This lesson extends the use of expected value to comparing strategies in a variety of contexts.

Classwork

Example 1 (2 minutes)

This example gives three probability distributions. Ask your students to verify that they are probability distributions (i.e., in each case, the probabilities add to 1). Explain that the club is going to sell only one product this year. Then ask students to share with their neighbor which product they think the club should sell based on an initial examination of the probability distributions.

MP.3

Example 1

A math club has been conducting an annual fundraiser for many years that involves selling products. The club advisors have kept records of revenue for the products that they have made and sold over the years and have constructed the following probability distributions for the three most popular products. (Revenue has been rounded to the nearest hundred dollars.)

Candy			Magazine Subscriptions			Wrapping Paper	
Revenue	Probability		Revenue	Probability		Revenue	Probability
$\$100.00$	0.10						
$\$200.00$	0.10		$\$200.00$	0.4			
$\$300.00$	0.25		$\$300.00$	0.4		$\$300.00$	1.0
$\$400.00$	0.45		$\$400.00$	0.2			
$\$500.00$	0.05						
$\$600.00$	0.05						

Scaffolding:

For advanced learners, consider combining Exercises 1 and 2 and asking students to determine which product should be offered.

- The club advisors only want to offer one product this year. They have decided to let the club members choose which product to offer and shared the records from past years. The overhead costs are $\$80.00$ for candy, $\$20.00$ for magazine subscriptions, and $\$40.00$ for wrapping paper. Assuming that these probability distributions were to hold for the coming fundraiser, which product should the club members recommend they sell? Explain.

For struggling students, consider presenting only two products as possible options for the fundraiser.

- The math club must choose between selling candy and magazine subscriptions. Which product should the members recommend?

Exercises 1–2 (5 minutes)

Have students work with a partner or in a small group on Exercises 1 and 2. As students complete the exercises, rotate around the room to informally assess their work. Then discuss and confirm answers as a class.

Exercises 1–2

1. **The club advisors only want to offer one product this year. They have decided to let the club members choose which product to offer and have shared the records from past years. Assuming that these probability distributions were to hold for the coming fundraiser, which product should the club members recommend they sell? Explain.**

   The expected revenues for the three products are

   Candy
   $(100)(0.10) + (200)(0.10) + (300)(0.25) + (400)(0.45) + (500)(0.05) + (600)(0.05) = \$340.00.$

   (Be sure your students include the units and dollars, and not just the raw number 340*.)*

   Magazine subscriptions
   $(200)(0.4) + (300)(0.4) + (400)(0.2) = \$280.00.$

   Wrapping paper
   $(300)(1) = \$300.00.$

   Because the expected revenue is the highest for candy, it is recommended that the math club sell this product.

2. **The club advisors forgot to include overhead costs with the past revenue data. The overhead costs are $\$80.00$ for candy, $\$20.00$ for magazine subscriptions, and $\$40.00$ for wrapping paper. Will this additional information change the product that the math club members recommend they sell? Why?**

   The math club's decision should not be based solely on expected revenue. Expenses have been incurred in producing the product. Such expenses are referred to as overhead costs. The result of revenue minus overhead cost yields profit. Based on expected profit, the math club can randomly choose one of the products since they all yield the same expected profit.

   The expected profit for producing candy is $\$340.00 - \$80.00 = \$260.00.$

   The expected profit for producing magazine subscriptions is $\$280.00 - \$20.00 = \$260.00.$

   The expected profit for producing wrapping paper is $\$300.00 - \$40.00 = \$260.00.$

Scaffolding:

For students who are struggling or for English language learners, consider displaying the following on the front board of the classroom to help students answer Exercise 2.

- *Overhead costs* are expenses that have been incurred to produce a product.
- The profit from the fundraiser is the result of the revenue minus the overhead costs:
  Profit = revenue – overhead.

Example 2 (2 minutes)

The game described in this example is a very simple version of a real game on television called *Deal or No Deal*. You may want to demonstrate the game by putting four boxes on a front table in your classroom and have one of your students play the role of wanting to buy a $7,500.00 pre-owned car. This example and the following exercises provide students with an opportunity to practice making decisions and justifying their decisions using expected value.

Example 2

A game on television has the following rules. There are four identical boxes on a table. One box contains $\$1.00$, the second, $\$15.00$, the third, $\$15,000.00$; and the fourth, $\$40,000.00$. You are offered the choice between taking $\$5,000.00$ or taking the amount of money in one of the boxes you choose at random.

Exercises 3–4 (5 minutes)

MP.3

Have students work with a partner to complete the exercises. After students have had time to think about the questions in Exercises 3 and 4, discuss the answers as a class. In both Exercise 3 and 4, students should use expected value to support their strategy. Emphasize that there is not a right answer to these questions. Teachers should look for students to justify their answers using expected value.

Exercises 3–4

3. **Suppose that you want to buy a $\$7,500.00$ pre-owned car. What should you do? Take the $\$5,000.00$ or choose a box? Why?**

   Answers will vary. Some students may say to take the $\$5,000.00$ since it is guarenteed money rather than potential money. But what if there is no way to earn the remaining $\$2,500.00$? Hopefully, a student will argue that a decision should be made based on the expected winning by playing the game. By choosing a box at random, the expected winning is $(1)\left(\frac{1}{4}\right)+(15)\left(\frac{1}{4}\right)+(15,000)\left(\frac{1}{4}\right)+(40,000)\left(\frac{1}{4}\right)=\$13,754.00$. That would be enough to buy the car, plus insurance and gas for an extended period of time.

4. **What should you do if you want to buy a $\$20,000.00$ brand-new car? Take the $\$5,000.00$ or choose a box? Why?**

   Just as in Exercise 3, there really isn't a right or wrong answer to the question posed in Exercise 4. But it will be interesting to hear students' arguments on both sides.

Example 3 (3 minutes)

Read through the example as a class. Chess is an example of a game for which this example could apply. This example differs from the others in this lesson as students must determine several different probability distributions based upon the different combination of strategies for games 1 and 2.

Note that you may have to review the multiplication rule that says that the probability of the intersection of two independent events is the product of the event probabilities. Henry chooses a strategy for game 2 independently of his choice for game 1 and whether he won, tied, or lost game 1. So probabilities are being multiplied in the following calculations.

Example 3

In a certain two-player game, players accumulate points. One point is earned for a win, half a point is earned for a tie, and zero points are earned for a loss. A match consists of two games. There are two different approaches for how the game can be played—boldly (B) or conservatively (C). Before a match, Henry wants to determine whether to play both games boldly (BB), one game boldly and one game conservatively (BC or CB), or both games conservatively (CC). Based on years of experience, he has determined the following probabilities for a win, a tie, or a loss depending on whether he plays boldly or conservatively.

Approach	Win (W)	Tie (T)	Lose (L)
Bold (B)	0.45	0.0	0.55
Conservative (C)	0.0	0.8	0.2

How should Henry play to maximize the expected number of points earned in the match? The following exercises will help you answer this question.

Exercises 5–14 (20 minutes)

Have students work together in small groups to complete the following exercises. To be sure that all students understand how points are earned, work through Exercise 5 as a class.

MP.3 For Exercises 6–13, there are four possible strategies: play boldly in both games (BB), play conservatively in both games (CC), play boldly in the first game and conservatively in the second (BC), play conservatively in the first game and boldly in the second (CB). You may want to assign one strategy per group of students. As students work through the exercises, rotate around the room to be sure that their tables are correct. Once the expected number of points earned in the match are computed for each strategy, bring the class together and discuss answers to Exercise 14 where students are asked to make a recommendation to Henry on how he should play. Students may have initially thought that playing boldly in both games would yield a larger expected number of points, when in fact the number is fairly close between all of the strategies. Their answers to Exercise 14 should include expected value as rationale for choosing the best strategy.

Exercises 5–14

5. **What are the possible values for the total points Henry can earn in a match? For example, he can earn $1\frac{1}{2}$ points by WT or TW (he wins the first game and ties the second game or ties the first game and wins the second game). What are the other possible values?**

   The possible numbers of points earned in a match are the entries in the following matrix. The possibilities are 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2 as shown in the following table.

		Game 2		
		Win (1)	*Tie* $\left(\frac{1}{2}\right)$	*Lose* (0)
Game 1	*Win* (1)	2	$1\frac{1}{2}$	1
	Tie $\left(\frac{1}{2}\right)$	$1\frac{1}{2}$	1	$\frac{1}{2}$
	Lose (0)	1	$\frac{1}{2}$	0

6. If Henry plays both games boldly (BB), find the probability that Henry will earn

   a. 2 points 0.2025

   b. $1\frac{1}{2}$ points 0.0

   c. 1 point 0.495

   d. $\frac{1}{2}$ point 0.0

   e. 0 points 0.3025

   If Henry plays both games boldly, the following table gives the possible points earned and the probabilities (printed below the points).

		Game 2 BOLD		
		Win (1) 0.45	Tie $\left(\frac{1}{2}\right)$ 0.0	Lose (0) 0.55
Game 1 BOLD	Win (1) 0.45	2 $(0.45)(0.45) = 0.2025$	$1\frac{1}{2}$ 0.0	1 $(0.45)(0.55) = 0.2475$
	Tie $\left(\frac{1}{2}\right)$ 0.0	$1\frac{1}{2}$ 0.0	1 0.0	$\frac{1}{2}$ 0.0
	Lose (0) 0.55	1 $(0.55)(0.45) = 0.2475$	$\frac{1}{2}$ 0.0	0 $(0.55)(0.55) = 0.3025$

7. What is the expected number of points that Henry will earn if he plays using a BB strategy?

   Henry's expected number of points won playing BB $= (2)(0.2025) + (1)(0.2475) + (1)(0.2475) + (0)(0.3025) = 0.9$ *points. (Be sure your students include the units.)*

8. If Henry plays both games conservatively (CC), find the probability that Henry will earn

   a. 2 points 0.0

   b. $1\frac{1}{2}$ points 0.0

   c. 1 point 0.64

   d. $\frac{1}{2}$ point 0.32

   e. 0 points 0.04

   If Henry plays both games conservatively, the following table gives the possible points earned and the probabilities (printed below the points).

		Game 2 CONSERVATIVE		
		Win (1) 0.0	Tie $\left(\frac{1}{2}\right)$ 0.8	Lose (0) 0.2
Game 1 CONS.	Win (1) 0.0	2 0.0	$1\frac{1}{2}$ 0.0	1 0.0
	Tie $\left(\frac{1}{2}\right)$ 0.8	$1\frac{1}{2}$ 0.0	1 $(0.8)(0.8) = 0.64$	$\frac{1}{2}$ $(0.8)(0.2) = 0.16$
	Lose (0) 0.2	1 0.0	$\frac{1}{2}$ $(0.2)(0.8) = 0.16$	0 $(0.2)(0.2) = 0.04$

9. What is the expected number of points that Henry will earn if he plays using a CC strategy?

Henry's expected number of points won playing CC $= (1)(0.64) + \left(\frac{1}{2}\right)(0.16) + \left(\frac{1}{2}\right)(0.16) + (0)(0.04) = 0.8$ *points.*

10. If Henry plays the first game boldly and the second game conservatively (BC), find the probability that Henry will earn

a. 2 points 0.0

b. $1\frac{1}{2}$ points 0.36

c. 1 point 0.09

d. $\frac{1}{2}$ point 0.44

e. 0 points 0.11

If Henry plays the first game boldly and the second game conservatively, the following table gives the possible points earned and the probabilities (printed below the points).

		Game 2 *CONSERVATIVE*		
		Win (1) 0.0	*Tie* $\left(\frac{1}{2}\right)$ 0.8	*Lose* (0) 0.2
Game 1 *BOLD*	*Win* (1) 0.45	2 0.0	$1\frac{1}{2}$ $(0.45)(0.8) = 0.36$	1 $(0.45)(0.2) = 0.09$
	Tie $\left(\frac{1}{2}\right)$ 0.0	$1\frac{1}{2}$ 0.0	1 0.0	$\frac{1}{2}$ 0.0
	Lose (0) 0.55	1 0.0	$\frac{1}{2}$ $(0.55)(0.8) = 0.44$	0 $(0.55)(0.2) = 0.11$

11. What is the expected number of points that Henry will earn if he plays using a BC strategy?

Henry's expected number of points won playing BC $= \left(1\frac{1}{2}\right)(0.36) + (1)(0.09) + \left(\frac{1}{2}\right)(0.44) + (0)(0.11) =$ 0.85 *points.*

12. **If Henry plays the first game conservatively and the second game boldly (CB), find the probability that Henry will earn**

   a. 2 points $\mathbf{0.0}$

   b. $1\frac{1}{2}$ points $\mathbf{0.36}$

   c. 1 point $\mathbf{0.09}$

   d. $\frac{1}{2}$ point $\mathbf{0.44}$

   e. 0 points $\mathbf{0.11}$

   Note: Some students may see the symmetry between BC and CB and conclude without calculation that CB will have an expected number of points equal to 0.85 points.

   If Henry plays the first game conservatively and the second game boldly, the following table gives the possible points earned and the probabilities (printed below the points).

		Game 2 BOLD		
		Win (1) 0.45	*Tie* $\left(\frac{1}{2}\right)$ 0.0	*Lose* (0) 0.55
Game 1 CONS	*Win* (1) 0.0	2 0.0	$1\frac{1}{2}$ 0.0	1 0.0
	Tie $\left(\frac{1}{2}\right)$ 0.8	$1\frac{1}{2}$ $(0.8)(0.45) = 0.36$	1 0.0	$\frac{1}{2}$ $(0.8)(0.55) = 0.44$
	Lose (0) 0.2	1 $(0.2)(0.45) = 0.09$	$\frac{1}{2}$ 0.0	0 $(0.2)(0.55) = 0.11$

13. **What is the expected number of points that Henry will earn if he plays using a CB strategy?**

   Henry's expected number of points won playing CB = $\left(1\frac{1}{2}\right)(0.36) + (1)(0.09) + \left(\frac{1}{2}\right)(0.44) + (0)(0.11) =$ 0.85 ***points.***

14. **Of the four possible strategies, which should Henry play in order to maximize his expected number of points earned in a match?**

   Henry's best strategy is to play both games boldly (expected points = 0.9). Playing one game boldly and the other conservatively yields an expected 0.85 points, whereas playing both games conservatively is the least desirable strategy, 0.8 points.

Closing (2 minutes)

- Explain how to use expected value to make decisions.
  - *Sample response. Expected value can be used to determine the best option for maximizing or minimizing an objective.*
- Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this as an opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

Lesson Summary

- **Making decisions in the face of uncertainty is one of the primary uses of statistics.**
- **Expected value can be used as one way to decide which of two or more alternatives is best for either maximizing or mininizing an objective.**

Exit Ticket (6 minutes)

Name ______________________________ Date ______________

Lesson 15: Using Expected Values to Compare Strategies

Exit Ticket

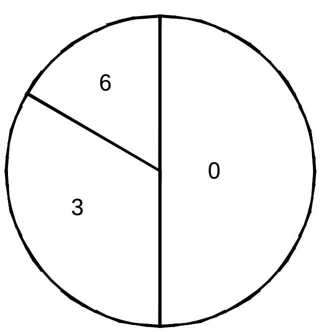
1. Your older sister asks you which of two summer job opportunities she should take. She likes them both. Job A is self-employed; Job B works with a friend. The probability distributions for the amount of money that can be earned per day, X, follow. In Job B, money earned will be split evenly between your sister and her friend.

Self-Employed Job A	
X	Probability
20	0.2
30	0.4
35	0.3
45	0.1

With Friend Job B	
X	Probability
50	0.3
75	0.6
100	0.1

Which opportunity would you recommend that your sister pursue? Explain why in terms of expected value.

2. A carnival game consists of choosing to spin the following spinner once or roll a pair of fair number cubes once. If the spinner lands on 0, you get no points; if it lands on 3, you get 3 points; if it lands on 6, you get 6 points. If the two number cubes sum to a prime number, you get 4 points. If the sum is not a prime number, you get 0 points. Should you spin the spinner or choose the number cubes if you want to maximize the expected number of points obtained? Explain why or why not. Note: The spinner is broken up into wedges representing $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$.



Exit Ticket Sample Solutions

1. Your older sister asks you which of two summer job opportunities she should take. She likes them both. Job A is self-employed; Job B works with a friend. The probability distributions for the amount of money that can be earned per day, X, follow. In Job B, money earned will be split evenly between your sister and her friend.

Self-Employed Job A	
X	Probability
20	0.2
30	0.4
35	0.3
45	0.1

With Friend Job B	
X	Probability
50	0.3
75	0.6
100	0.1

Which opportunity would you recommend that your sister pursue? Explain why in terms of expected value.

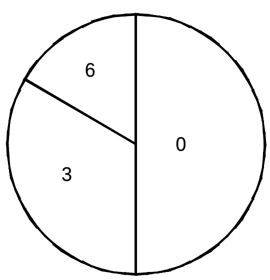
(Be sure your students first check that the probabilities add to 1 in each case.) Since your sister has no preference regarding the jobs, your recommendation should be based on expected earnings.

$E(\text{earnings for Job A}) = (20)(0.2) + (30)(0.4) + (35)(0.3) + (45)(0.1) = \$31.00$ *earnings per day*

$E(\text{earnings for Job B}) = (50)(0.3) + (75)(0.6) + (100)(0.1) = \$70.00$ *earnings per day for two people, so* $\$35.00$ *per day for each*

Recommend Job B because the expected earnings per day are $\$35.00$*, which is more than the expected earnings for Job A.*

2. A carnival game consists of choosing to spin the following spinner once or roll a pair of fair number cubes once. If the spinner lands on 0, you get no points; if it lands on 3, you get 3 points; if it lands on 6, you get 6 points. If the two number cubes sum to a prime number, you get 4 points. If the sum is not a prime number, you get 0 points. Should you spin the spinner or choose the number cubes if you want to maximize the expected number of points obtained? Explain why or why not. Note: The spinner is broken up into wedges representing $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{6}$.



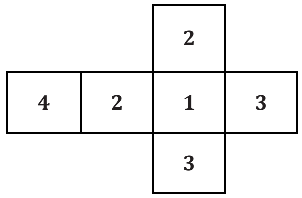
The expected number of points spinning the spinner is $(0)\left(\frac{1}{2}\right) + (3)\left(\frac{2}{6}\right) + (6)\left(\frac{1}{6}\right) = 2$ *points.*

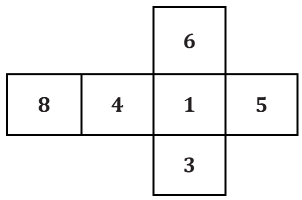
The expected number of points rolling the fair number cubes is $(4)\left(\frac{15}{36}\right) = 1\frac{2}{3}$ *points.*

Spin the spinner because the expected number of points from spinning the spinner is slightly higher than the expected number of points from rolling the fair number cubes.

Problem Set Sample Solutions

1. A game allows you to choose what number cubes you would like to use to play. One pair of number cubes is a regular pair in which the sides of each cube are numbered from 1 to 6. The other pair consists of two different cubes as shown below. For all of these number cubes, it is equally likely that the cube will land on any one of its six sides.





a. Suppose that you want to maximize the expected sum per roll in the long run. Which pair of number cubes should you use? Explain why.

For the regular fair number cubes, the expected sum per roll is $(2)\left(\frac{1}{36}\right)+(3)\left(\frac{2}{36}\right)+(4)\left(\frac{3}{36}\right)+(5)\left(\frac{4}{36}\right)+(6)\left(\frac{5}{36}\right)+(7)\left(\frac{6}{36}\right)+(8)\left(\frac{5}{36}\right)+(9)\left(\frac{4}{36}\right)+(10)\left(\frac{3}{36}\right)+(11)\left(\frac{2}{36}\right)+(12)\left(\frac{1}{36}\right)=7.$

For the Sicherman number cubes, first note that the distribution of the sum is (surprisingly) the same as that for the regular number cubes. The expected sum per roll is 7.

	1	3	4	5	6	8
1	2	4	5	6	7	9
2	3	5	6	7	8	10
2	3	5	6	7	8	10
3	4	6	7	8	9	11
3	4	6	7	8	9	11
4	5	7	8	9	10	12

Since the expected sum per roll is 7 *for each pair of cubes, you could choose either pair of number cubes.*

b. Imagine that you are playing a game in which you earn special privileges by rolling doubles (i.e., the same number on both cubes). Which number cubes would you prefer to use? Explain.

Students should then be able to suggest that the regular number cubes have 6 *possible doubles, but the Sicherman number cubes have only* 4*. So, for games that have special rules for rolling doubles, the regular number cubes are preferable despite the fact that the expected sum per roll is the same for each type of cube.*

Note on part (b): Whereas it doesn't matter which pair of number cubes is used if the game's rules were based strictly on the sum with no added conditions, ask your students if it would matter which pair were used to play Monopoly. You may have to tell students that in the rules of Monopoly there are special moves that can be made if the roll results in a pair (called rolling doubles). Students should then be able to suggest that the regular number cubes have 6 possible pairs but the Sicherman number cubes have only 4. So, for games that have special rules for pairs, the regular number cubes are preferable despite the fact that the expected sum per roll is the same.

2. Amy is a wedding planner. Some of her clients care about whether the wedding is held indoors or outdoors depending on weather conditions as well as respective costs. Over the years, Amy has compiled the following data for June weddings. (Costs are in thousands of dollars.)

Weather	Cost Indoors	Cost Outdoors	Probability
Cold and sunny	$\$29$	$\$33$	0.15
Cold and rainy	$\$30$	$\$40$	0.05
Warm and sunny	$\$22$	$\$27$	0.45
Warm and rainy	$\$24$	$\$30$	0.35

a. What is the expected cost of a June wedding held indoors?

Note: Be sure your students check that the probabilities add to 1.

The expected cost of a June wedding held indoors is $(29)(0.15) + (30)(0.05) + (22)(0.45) + (24)(0.35) = 24.15$ thousands of dollars, i.e., $\$24,150.00$.

b. What is the expected cost of a June wedding held outdoors?

The expected cost of a June wedding held outdoors is $(33)(0.15) + (40)(0.05) + (27)(0.45) + (30)(0.35) = 29.60$ thousands of dollars, i.e., $\$29,600.00$.

c. A new client has her heart set on an outdoor wedding. She has at most $\$25,000.00$ available. What do you think Amy told the client and why?

Amy would have told her new client that she has enough money to cover an indoor wedding, as the expected cost is $\$24,150.00$. However, no matter the weather conditions, the client will not be able to afford an outdoor wedding. The expected cost of an outdoor wedding (regardless of weather) is $\$29,600.00$, so the client's budget falls $\$4,600.00$ short.

3. A venture capitalist is considering two investment proposals. One proposal involves investing $\$100,000.00$ in a green alternative energy source. The probability that it will succeed is only 0.05, but the gain on investment would be $\$2,500,000.00$. The other proposal involves investing $\$300,000.00$ in an existing textile company. The probability that it will succeed is 0.5, and the gain on investment would be $\$725,000.00$. In which proposal should the venture capitalist invest? Explain.

The following are probability distributions for each proposal:

Green Energy	
Gain	*Probability*
$\$0.00$	0.95
$\$2,500,000.00$	0.05

Textile Company	
Gain	*Probability*
$\$0.00$	0.5
$\$725,000.00$	0.5

The expected gain for each proposal is as follows:

Green Energy: $0(0.95) + 2,500,000(0.05) = \$125,000.00$

Textile Company: $0(0.5) + 725,000(0.5) = \$362,500.00$

The venture capitalist's expected profit can be found by subtracting the amount invested from the expected gain.

The expected profit from the green energy proposal is $\$125,000.00 - \$100,000.00$, *or* $\$25,000.00$.

The expected profit from the textile proposal is $\$362,500.00 - \$300,000.00$, *or* $\$62,500.00$.

The venture capitalist should invest in the textile company because the expected profit is higher.

4. A student is required to purchase injury insurance in order to participate on his high school football team. The insurance will cover all expenses incurred if the student is injured during a football practice or game, but the student must pay a deductible for submitting a claim. There is also an up-front cost to purchase the injury insurance.

- Plan A costs $\$75.00$ up front. If the student is injured and files a claim, the deductible is $\$100.00$.
- Plan B costs $\$100.00$ up front. If the student is injured and files a claim, the deductible is $\$50.00$.

Suppose there is a 1 in 5 chance of the student making a claim on the insurance policy. Which plan should the student choose? Explain.

The probability distributions for each plan are as follows:

Plan A	
Deductible	*Probability*
$\$0.00$	$\frac{4}{5}$
$\$100.00$	$\frac{1}{5}$

Plan B	
Deductible	*Probability*
$\$0.00$	$\frac{4}{5}$
$\$50.00$	$\frac{1}{5}$

The expected cost of submitting a claim for each plan is as follows:

Plan A: $0\left(\frac{4}{5}\right) + 100\left(\frac{1}{5}\right) = \$20.00$

Plan B: $0\left(\frac{4}{5}\right) + 50\left(\frac{1}{5}\right) = \$10.00$

Because there is also an up-front cost to purchase the insurance policy, the up-front cost should be combined with the expected cost of submitting a claim to determine the total out-of-pocket expense for the student.

The expected total cost of Plan A is $\$20.00 + \$75.00$ *or* $\$95.00$. *The expected total cost of Plan B is* $\$10.00 + \$100.00$ *or* $\$110.00$.

The student should choose Plan A in order to minimize his out-of-pocket expenses.