Lesson 9: Determining Discrete Probability Distributions

Student Outcomes

- Given a description of a discrete random variable, students determine the probability distribution of that variable.

Lesson Notes

In this lesson, students are given a description of a chance experiment that results in a discrete random variable. Students derive the discrete probability distribution for that random variable, and use the discrete probability distribution to answer probability questions. It is important to be very specific with language. The discrete probability distribution is a mathematical calculation based on possible outcomes of an event. It does not say much about what would actually happen if the following experiments were attempted. A coin flipped 10 times could land on heads every time. A coin flipped $1{,}000$ times could land on heads every time. However, there could be some introductory discussion of the idea that the larger the number of times the event occurs, the closer to the discrete probability distribution the outcomes will be.

Classwork

Exercises 1–3 (10 minutes)

Students should complete Exercises 1–3 either independently or with a partner. Discuss the answers as a class when the students are done. As students work, take a look at what they are producing. As students' work is informally assessed, choose students to share their answers as part of the discussion process.

Scaffolding:

If students are struggling, consider using the following questions to guide them:

- What are the possible values for this random variable?
  - *The values for this random variable are 0, 1, and 2. In other words, 0 heads, 1 head, or 2 heads can be observed.*
- How many possible outcomes are there for the chance experiment of flipping a penny and a nickel? (Hint: Use the counting principle.)
  - *There are $2 \times 2 = 4$ possible outcomes.*

Exercises

A chance experiment consists of flipping a penny and a nickel at the same time. Consider the random variable of the number of heads observed.

MP.4

1. Create a discrete probability distribution for the number of heads observed.

Penny	Nickel	Calculation	Probability
H	*H*	0.5×0.5	0.25
H	*T*	0.5×0.5	0.25
T	*H*	0.5×0.5	0.25
T	*T*	0.5×0.5	0.25

Number of Heads	0	1	2
Probability	0.25	0.50	0.25

2. Explain how the discrete probability distribution is useful.

It can be used to help make predictions. For example, if the scenario presents a game where I could win a prize for guessing the correct number of heads, I would choose 1 head since the probability of only one *head appearing is 0.5.*

3. What is the probability of observing at least one head when you flip a penny and a nickel?

The probability of tossing at least one head is as follows

$$P(1\text{ head}) + P(2\text{ heads}) = 0.50 + 0.25 = 0.75$$

Scaffolding:

Consider having students above-grade level attempt the following extension to the lesson:

- Invent a scenario that requires the calculation of a probability distribution. (Perhaps provide a broad topic, such as *food* or *shoes* to help focus their brainstorming.)
- Their scenarios and probabilities do not necessarily need to be research-based (they could be fictional), but their probability distributions should be accurate for their scenarios.

Exercises 4–6 (12 minutes)

Students can work independently or with a partner to complete Exercises 4–6. Be sure that students complete the tables correctly. Discuss the answers when students are done. As students progress from Exercises 1–3 to Exercises 4–6, they should notice that, if outcomes are composed of j events, then there are 2^j possible outcomes. (This may be a good introductory question.) Also, they should understand that probabilities asking for *at least* or *at most* are calculated by addition. Since these outcomes are discrete, multiplication does not make sense. (It is not possible for two coins to come up both heads and both tails.)

MP.4

Suppose that on a particular island, 60% of the eggs of a certain type of bird are female. You spot a nest of this bird and find three eggs. You are interested in the number of male eggs. Assume the gender of each egg is independent of the other eggs in the nest.

4. Create a discrete probability distribution for the number of male eggs in the nest.

Egg 1	*Egg 2*	*Egg 3*	*Calculation*	*Probability*
F	*F*	*F*	$0.6 \times 0.6 \times 0.6$	0.216
F	*F*	*M*	$0.6 \times 0.6 \times 0.4$	0.144
F	*M*	*F*	$0.6 \times 0.4 \times 0.6$	0.144
F	*M*	*M*	$0.6 \times 0.4 \times 0.4$	0.096
M	*F*	*F*	$0.4 \times 0.6 \times 0.6$	0.144
M	*F*	*M*	$0.4 \times 0.6 \times 0.4$	0.096
M	*M*	*F*	$0.4 \times 0.4 \times 0.6$	0.096
M	*M*	*M*	$0.4 \times 0.4 \times 0.4$	0.064

Number of Male Eggs	0	1	2	3
Probability	0.216	0.432	0.288	0.064

5. What is the probability that no more than two eggs are male?

The probability that no more than two eggs are male is as follows

$$0.216 + 0.432 + 0.288 = 0.936$$

6. **Explain the similarities and differences between this probability distribution and the one in the first part of the lesson.**

The distributions are similar because the events are independent in both cases, so the probability of each outcome can be determined by multiplying the probabilities of the events. The distributions are different because the number of values for each of the random variables is different.

Exercise 7 (15 minutes)

Begin work on this exercise as a whole class. Assign various students to perform the individual probability calculations. Then, combine class results and have students complete the remainder of the exercise independently. Consider asking students what they think it means to be a *satisfied customer*.

7. **The manufacturer of a certain type of tire claims that only 5% of the tires are defective. All four of your tires need to be replaced. What is the probability you would be a satisfied customer if you purchased all four tires from this manufacturer? Would you purchase from this manufacturer? Explain your answer using a probability distribution.**

Tire 1	*Tire 2*	*Tire 3*	*Tire 4*	*Calculation*	*Probability*
D	*D*	*D*	*D*	$0.05 \times 0.05 \times 0.05 \times 0.05$	0.00000625
D	*D*	*D*	*ND*	$0.05 \times 0.05 \times 0.05 \times 0.95$	0.00011875
D	*D*	*ND*	*D*	$0.05 \times 0.05 \times 0.95 \times 0.05$	0.00011875
D	*ND*	*D*	*D*	$0.05 \times 0.95 \times 0.05 \times 0.05$	0.00011875
D	*D*	*ND*	*ND*	$0.05 \times 0.05 \times 0.95 \times 0.95$	0.00225625
D	*ND*	*D*	*ND*	$0.05 \times 0.95 \times 0.05 \times 0.95$	0.00225625
D	*ND*	*ND*	*D*	$0.05 \times 0.95 \times 0.95 \times 0.05$	0.00225625
D	*ND*	*ND*	*ND*	$0.05 \times 0.95 \times 0.95 \times 0.95$	0.04286875
ND	*D*	*D*	*D*	$0.95 \times 0.05 \times 0.05 \times 0.05$	0.00011875
ND	*D*	*D*	*ND*	$0.95 \times 0.05 \times 0.05 \times 0.95$	0.00225625
ND	*D*	*ND*	*D*	$0.95 \times 0.05 \times 0.95 \times 0.05$	0.00225625
ND	*ND*	*D*	*D*	$0.95 \times 0.95 \times 0.05 \times 0.05$	0.00225625
ND	*D*	*ND*	*ND*	$0.95 \times 0.05 \times 0.95 \times 0.95$	0.04286875
ND	*ND*	*D*	*ND*	$0.95 \times 0.95 \times 0.05 \times 0.95$	0.04286875
ND	*ND*	*ND*	*D*	$0.95 \times 0.95 \times 0.95 \times 0.05$	0.04286875
ND	*ND*	*ND*	*ND*	$0.95 \times 0.95 \times 0.95 \times 0.95$	0.81450625

Note: D stands for "defective," and ND stands for "not defective."

Number of Defective Tires	0	1	2	3	4
Probability	0.81450625	0.171475	0.0135375	0.000475	0.00000625

To be a satisfied customer, 0 tires would be defective. The probability of this happening is 0.81450625.

Answers will vary about whether students would purchase tires from this manufacturer, but this could lead to a discussion about whether this probability is "good enough."

Some sample discussion points:

- *81% is fairly high, but that means that there is a 19% chance that at least one tire is defective.*
- *According to this model, about 1 in 5 cars is expected to have at least one defective tire.*
- *Going further: Why is the probability for a vehicle to have at least one defective tire so much higher than the 5% defect rate? Is the advertised defect rate of 5% misleading?*

Closing (3 minutes)

- Ask students to explain the concept in writing and share their answer with a neighbor: Explain what a discrete probability distribution is and how is it useful.
    - *Sample response: A discrete probability distribution is the set of calculated probabilities of every possible outcome of a series of events. It is useful because it allows us to make predictions about what might happen if we attempted to actually test these events. It is also useful when determining the best course of action to take, as in a game, for instance.*
- To calculate the corresponding probabilities for the values of a random variable, add the individual probabilities for all outcomes that correspond to the value.
- Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this as an opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

Lesson Summary

- **To derive a probability distribution for a discrete random variable, you must consider all possible outcomes of the chance experiment.**
- **A discrete probability distribution displays all possible values of a random variable and the corresponding probabilities.**

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 9: Determining Discrete Probability Distributions

Exit Ticket

Suppose that an estimated 10% of the inhabitants of a large island have a certain gene. If pairs of islanders are selected at random and tested for the gene, what is the probability that one or both islanders are carriers? Explain your answer using a probability distribution.

Exit Ticket Sample Solutions

Suppose that an estimated 10% of the inhabitants of a large island have a certain gene. If pairs of islanders are selected at random and tested for the gene, what is the probability that one or both islanders are carriers? Explain your answer using a probability distribution.

Person 1	*Person 2*	*Calculation*	*Probability*
Y	*Y*	0.10×0.10	0.01
Y	*N*	0.10×0.90	0.09
N	*Y*	0.90×0.10	0.09
N	*N*	0.90×0.90	0.81

Number of Islanders with the Gene	0	1	2
Probability	0.81	0.18	0.01

The probability that one or both islanders are carriers is as follows

$$0.18 + 0.01 = 0.19$$

Problem Set Sample Solutions

1. **About 11% of adult Americans are left-handed. Suppose that two people are randomly selected from this population.**

   a. **Create a discrete probability distribution for the number of left-handed people in a sample of two randomly selected adult Americans.**

Person 1	*Person 2*	*Calculation*	*Probability*
L	*L*	0.11×0.11	0.0121
L	*R*	0.11×0.89	0.0979
R	*L*	0.89×0.11	0.0979
R	*R*	0.89×0.89	0.7921

Note: L stands for "left-handed," and R stands for "right-handed"

Number of Left-handers	0	1	2
Probability	0.7921	0.1958	0.0121

   b. **What is the probability that at least one person in the sample is left-handed?**

   The probability that at least one person is left-handed is as follows

$$0.1958 + 0.0121 = 0.2079$$

2. In a large batch of M&M candies, about 24% of the candies are blue. Suppose that three candies are randomly selected from the large batch.

   a. Create a discrete probability distribution for the number of blue candies out of the three randomly selected candies.

Candy 1	Candy 2	Candy 3	Calculation	Probability
B	*B*	*B*	$0.24 \times 0.24 \times 0.24$	0.013824
B	*B*	*NB*	$0.24 \times 0.24 \times 0.76$	0.043776
B	*NB*	*B*	$0.24 \times 0.76 \times 0.24$	0.043776
B	*NB*	*NB*	$0.24 \times 0.76 \times 0.76$	0.138624
NB	*B*	*B*	$0.76 \times 0.24 \times 0.24$	0.043776
NB	*B*	*NB*	$0.76 \times 0.24 \times 0.76$	0.138624
NB	*NB*	*B*	$0.76 \times 0.76 \times 0.24$	0.138624
NB	*NB*	*NB*	$0.76 \times 0.76 \times 0.76$	0.438976

Note: B stands for "blue candy," and NB stands for "not blue candy."

Number of Blue Candies	0	1	2	3
Probability	0.438976	0.415872	0.131328	0.013824

   b. What is probability that at most two candies are blue? Explain how you know.

   The probability that at most two candies are blue is as follows:

$$0.438976 + 0.415872 + 0.131328 = 0.986176$$

3. In the 21st century, about 3% of mothers give birth to twins. Suppose three mothers-to-be are chosen at random.

   a. Create a discrete probability distribution for the number of sets of twins born from the sample.

Mother 1	Mother 2	Mother 3	Calculation	Probability
T	*T*	*T*	$0.03 \times 0.03 \times 0.03$	0.000027
T	*T*	*NT*	$0.03 \times 0.03 \times 0.97$	0.000873
T	*NT*	*T*	$0.03 \times 0.97 \times 0.03$	0.000873
T	*NT*	*NT*	$0.03 \times 0.97 \times 0.97$	0.028227
NT	*T*	*T*	$0.97 \times 0.03 \times 0.03$	0.000873
NT	*T*	*NT*	$0.97 \times 0.03 \times 0.97$	0.028227
NT	*NT*	*T*	$0.97 \times 0.97 \times 0.03$	0.028227
NT	*NT*	*NT*	$0.97 \times 0.97 \times 0.97$	0.912673

Note: T stands for "twins," and NT stands for "not twins."

Number of Mothers who Have Twins	0	1	2	3
Probability	0.912673	0.084681	0.002619	0.000027

   b. What is the probability that at least one of the three mothers did not give birth to twins?

$$0.912673 + 0.084681 + 0.002619 = 0.999973$$

4. About three in 500 people have type O-negative blood. Though it is one of the least frequently-occurring blood types, it is one of the most sought after because it can be donated to people who have any blood type.

   a. Create a discrete probability distribution for the number of people who have type O-negative blood in a sample of two randomly selected adult Americans.

Person 1	Person 2	Calculation	Probability
O	*O*	0.006×0.006	0.000036
O	*NO*	0.006×0.994	0.005964
NO	*O*	0.994×0.006	0.005964
NO	*NO*	0.994×0.994	0.988036

Note: O stands for "O-negative," and NO stands for "not O-negative."

Number of People with Type O-Negative Blood	0	1	2
Probability	0.988036	0.011928	0.000036

   b. Suppose two samples of two people are taken. What is the probability that at least one person in each sample has type O-negative blood?

$$(0.011928 + 0.000036)^2 = 0.000143$$

5. The probability of being struck by lightning in one's lifetime is approximately 1 in $3,000$.

   a. What is the probability of being struck by lightning twice in one's lifetime?

$$(0.000333)^2 = 0.000000111$$

   b. In a random sample of three adult Americans, how likely is it that at least one has been struck by lightning exactly twice?

Person 1	Person 2	Person 3	Calculation	Probability
T	*T*	*T*	$0.000000111 \times 0.000000111 \times 0.000000111$	1.3676×10^{-21}
T	*T*	*NT*	$0.000000111 \times 0.000000111 \times 0.999999889$	1.2321×10^{-14}
T	*NT*	*T*	$0.000000111 \times 0.999999889 \times 0.000000111$	1.2321×10^{-14}
T	*NT*	*NT*	$0.000000111 \times 0.999999889 \times 0.999999889$	1.1099×10^{-7}
NT	*T*	*T*	$0.999999889 \times 0.000000111 \times 0.000000111$	1.2321×10^{-14}
NT	*T*	*NT*	$0.999999889 \times 0.000000111 \times 0.999999889$	1.1099×10^{-7}
NT	*NT*	*T*	$0.999999889 \times 0.999999889 \times 0.000000111$	1.1099×10^{-7}
NT	*NT*	*NT*	$0.999999889 \times 0.999999889 \times 0.999999889$	0.999999667

Note: T stands for "twice," and NT stands for "not twice."

Number of People Struck Twice by Lightning	0	1	2	3
Probability	0.999999667	3.3297×10^{-7}	3.6963×10^{-14}	1.3676×10^{-21}

The probability that at least one person has been struck exactly twice by lightning is as follows:
$3.3297 \times 10^{-7} + 3.6963 \times 10^{-14} + 1.3676 \times 10^{-21} = 3.32970037 \times 10^{-7}$