

Lesson 4: Using Permutations and Combinations to Compute Probabilities

Classwork

Exercises 1–6

1. A high school is planning to put on the musical *West Side Story*. There are 20 singers auditioning for the musical. The director is looking for two singers who could sing a good duet. In how many ways can the director choose two singers from the 20 singers?

Indicate if this question involves a permutation or a combination. Give a reason for your answer.

2. The director is also interested in the number of ways to choose a lead singer and a backup singer. In how many ways can the director choose a lead singer and then a backup singer?

Indicate if this question involves a permutation or a combination. Give a reason for your answer.

3. For each of the following, indicate if it is a problem involving permutations, combinations, or neither, and then answer the question posed. Explain your reasoning.

- a. How many groups of five songs can be chosen from a list of 35 songs?

- b. How many ways can a person choose three different desserts from a dessert tray of eight desserts?

- c. How many ways can a manager of a baseball team choose the lead-off batter and second batter from a baseball team of nine players?

- d. How many ways are there to place seven distinct pieces of art in a row?
- e. How many ways are there to randomly select four balls without replacement from a container of 15 balls numbered 1 to 15?
4. The manager of a large store that sells TV sets wants to set up a display of all the different TV sets that they sell. The manager has seven different TVs that have screen sizes between 37 and 43 inches, nine that have screen sizes between 46 and 52 inches, and twelve that have screen sizes of 55 inches or greater.
- a. In how many ways can the manager arrange the 37–43 inch TV sets?
- b. In how many ways can the manager arrange the 55-inch or greater TV sets?
- c. In how many ways can the manager arrange all the TV sets if he is concerned about the order they were placed in?
5. Seven slips of paper with the digits 1 to 7 are placed in a large jar. After thoroughly mixing the slips of paper, two slips are picked without replacement.
- a. Explain the difference between ${}_7P_2$ and ${}_7C_2$ in terms of the digits selected.

- b. Describe a situation in which ${}_7P_2$ is the total number of outcomes.
- c. Describe a situation in which ${}_7C_2$ is the total number of outcomes.
- d. What is the relationship between ${}_7P_2$ and ${}_7C_2$?
6. If you know ${}_nC_k$, and you also know the value of n and k , how could you find the value of ${}_nP_k$? Explain your answer.

Example 1: Calculating Probabilities

In a high school there are 10 math teachers. The principal wants to form a committee by selecting three math teachers at random. If Mr. H, Ms. B, and Ms. J are among the group of 10 math teachers, what is the probability that all three of them will be on the committee?

Because every different committee of 3 is equally likely,

$$P(\text{these three math teachers will be on the committee}) = \frac{\text{number of ways Mr. H, Ms. B, and Ms. J can be selected}}{\text{total number of 3 math teacher committees that can be formed}}$$

The total number of possible committees is the number of ways that three math teachers can be chosen from 10 math teachers, which is the number of combinations of 10 math teachers taken 3 at a time or ${}_{10}C_3 = 120$. Mr. H, Ms. B, and Ms. J form one of these selections. The probability that the committee will consist of Mr. H, Ms. B, and Ms. J is $\frac{1}{120}$.

Exercises 7–9

7. A high school is planning to put on the musical *West Side Story*. There are 20 singers auditioning for the musical. The director is looking for two singers who could sing a good duet.
- What is the probability that Alicia and Juan are the two singers who are selected by the director? How did you get your answer?

 - The director is also interested in the number of ways to choose a lead singer and a backup singer. What is the probability that Alicia is selected the lead singer and Juan is selected the backup singer? How did you get your answer?

8. For many computer tablets, the owner can set a 4-digit pass code to lock the device.
- How many different 4-digit pass codes are possible if the digits cannot be repeated? How did you get your answer?
 - If the digits of a pass code are chosen at random and without replacement from the digits 0, 1, ..., 9, what is the probability that the pass code is 1234? How did you get your answer?
 - What is the probability that two people, who both chose a pass code by selecting digits at random and without replacement, both have a pass code of 1234? Explain your answer.
9. A chili recipe calls for ground beef, beans, green pepper, onion, chili powder, crushed tomatoes, salt, and pepper. You have lost the directions about the order in which to add the ingredients, so you decide to add them in a random order.
- How many different ways are there to add the ingredients? How did you get this answer?
 - What is the probability that the first ingredient that you add is crushed tomatoes? How did you get your answer?

- c. What is the probability that the ingredients are added in the exact order listed above? How did you get your answer?

Example 2: Probability and Combinations

A math class consists of 14 girls and 15 boys. The teacher likes to have the students come to the board to demonstrate how to solve some of the math problems. During a lesson the teacher randomly selects 6 of the students to show their work. What is the probability that all 6 of the students selected are girls?

$$P(\text{all 6 students are girls}) = \frac{\text{number of ways to select 6 girls out of 14}}{\text{number of groups of 6 from the whole class}}$$

The number of ways to select 6 girls from the 14 girls is the number of combinations of 6 from 14 which is ${}_{14}C_6 = 3,003$. The total number of groups of 6 is ${}_{29}C_6 = 475,020$.

The probability that all 6 students are girls is

$$P(\text{all 6 students are girls}) = \frac{{}_{14}C_6}{{}_{29}C_6} = \frac{3,003}{475,020} = 0.006$$

Exercises 10–11

10. There are nine golf balls numbered from 1 to 9 in a bag. Three balls are randomly selected without replacement to form a 3-digit number.
- How many 3-digit numbers can be formed? Explain your answer.

 - How many 3-digit numbers start with the digit 1? Explain how you got your answer.

- c. What is the probability that the 3-digit number formed is less than 200? Explain your answer.
11. There are eleven seniors and five juniors who are sprinters on the high school track team. The coach must select four sprinters to run the 800-meter relay race.
- a. How many 4-sprinter relay teams can be formed from the group of 16 sprinters?
- b. In how many ways can two seniors be chosen to be part of the relay team?
- c. In how many ways can two juniors be chosen to be part of the relay team?
- d. In how many ways can two seniors and two juniors be chosen to be part of the relay team?
- e. What is the probability that two seniors and two juniors will be chosen for the relay team?

Lesson Summary

- The number of permutations of n things taken k at a time is

$${}_n P_k = \frac{n!}{(n-k)!}$$

- The number of combinations of k items selected from a set of n distinct items is

$${}_n C_k = \frac{{}_n P_k}{k!} \text{ or } {}_n C_k = \frac{n!}{k!(n-k)!}$$

- Permutations and combinations can be used to calculate probabilities.

Problem Set

1. For each of the following, indicate whether it is a question that involves permutations, combinations, or neither, and then answer the question posed. Explain your reasoning.
 - a. How many ways can a coach choose two co-captains from 16 players in the basketball team?
 - b. In how many ways can seven questions out of ten be chosen on an examination?
 - c. Find the number of ways that 10 women in the finals of the skateboard street competition can finish first, second, and third in the X Games final.
 - d. A postal zip code contains five digits. How many different zip codes can be made with the digits 0–9? Assume a digit can be repeated.
2. Four pieces of candy are drawn at random from a bag containing five orange pieces and seven brown pieces.
 - a. How many different ways can four pieces be selected from the 12 colored pieces?
 - b. How many different ways can two orange pieces be selected from five orange pieces?
 - c. How many different ways can two brown pieces be selected from seven brown pieces?
3. Consider the following:
 - a. A game was advertised as having a probability of 0.4 of winning. You know that the game involved five cards with a different digit on each card. Describe a possible game involving the cards that would have a probability of 0.4 of winning.
 - b. A second game involving the same five cards was advertised as having a winning probability of 0.05. Describe a possible game that would have a probability of 0.05 or close to 0.05 of winning
4. You have five people who are your friends on a certain social network. You are related to two of the people, but you do not recall who of the five people are your relatives. You are going to invite two of the five people to a special meeting. If you randomly select two of the five people to invite, explain how you would derive the probability of inviting your relatives to this meeting?

5. Charlotte is picking out her class ring. She can select from a ruby, an emerald, or an opal stone, and she can also select silver or gold for the metal.
- How many different combinations of one stone and one type of metal can she choose? Explain how you got your answer.
 - If Charlotte selects a stone and a metal at random, what is the probability that she would select a ring with a ruby stone and gold metal?
6. In a lottery, three numbers are chosen from 0 to 9. You win if the three numbers you pick match the three numbers selected by the lottery machine.
- What is the probability of winning this lottery if the numbers cannot be repeated?
 - What is the probability of winning this lottery if the numbers can be repeated?
 - What is the probability of winning this lottery if you must match the exact order that the lottery machine picked the numbers?
7. The store at your school wants to stock t-shirts that come in five sizes (small, medium, large, XL, XXL) and in two colors (orange and black).
- How many different type t-shirts will the store have to stock?
 - At the next basketball game, the cheerleaders plan to have a t-shirt toss. If they have one t-shirt of each type in a box and select a shirt at random, what is the probability that the first randomly selected t-shirt is a large orange t-shirt?
8. There are 10 balls in a bag numbered from 1 to 10. Three balls are selected at random without replacement.
- How many different ways are there of selecting the three balls?
 - What is the probability that one of the balls selected is the number 5?
9. There are nine slips of paper in a bag numbered from 1 to 9 in a bag. Four slips are randomly selected without replacement to form a 4-digit number.
- How many 4-digit numbers can be formed?
 - How many 4-digit numbers start with the digit 1?
 - What is the probability that the 2-digit number formed is less than 20?
10. There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference.
- How many different ways are there to select a group of four students to attend the conference from the 37 Service Club members?
 - How many ways are there to select exactly two juniors?
 - How many ways are there to select exactly two seniors?
 - If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?
 - What is the probability that two juniors and two seniors are selected to attend the conference?

11. A basketball team of 16 players consists of 6 guards, 7 forwards, and 3 centers. Coach decides to randomly select 5 players to start the game. What is the probability of 2 guards, 2 forwards, and 1 center starting the game?

12. A research study was conducted to estimate the number of white perch (a type of fish) in a Midwestern lake. 300 perch were captured and tagged. After they were tagged, the perch were released back into the lake. A scientist involved in the research estimates there are 1,000 perch in this lake. Several days after tagging and releasing the fish, the scientist caught 50 perch of which 20 were tagged. If this scientist's estimate about the number of fish in the lake is correct, do you think it was likely to get 20 perch out of 50 with a tag? Explain your answer.