Lesson 4: Using Permutations and Combinations to Compute Probabilities

Classwork

Exercises 1–6

1. A high school is planning to put on the musical *West Side Story*. There are singers auditioning for the musical. The director is looking for two singers who could sing a good duet. In how many ways can the director choose two singers from the singers?

Indicate if this question involves a permutation or a combination. Give a reason for your answer.

1. The director is also interested in the number of ways to choose a lead singer and a backup singer. In how many ways can the director choose a lead singer and then a backup singer?

Indicate if this question involves a permutation or a combination. Give a reason for your answer.

1. For each of the following, indicate if it is a problem involving permutations, combinations, or neither, and then answer the question posed. Explain your reasoning.
   1. How many groups of five songs can be chosen from a list of songs?
   2. How many ways can a person choose three different desserts from a dessert tray of eight desserts?
   3. How many ways can a manager of a baseball team choose the lead-off batter and second batter from a baseball team of nine players?
   4. How many ways are there to place seven distinct pieces of art in a row?
   5. How many ways are there to randomly select four balls without replacement from a container of balls numbered to ?
2. The manager of a large store that sells TV sets wants to set up a display of all the different TV sets that they sell. The manager has seven different TVs that have screen sizes between and inches, nine that have screen sizes between and inches, and twelve that have screen sizes of inches or greater.
   1. In how many ways can the manager arrange the – inch TV sets?
   2. In how many ways can the manager arrange the -inch or greater TV sets?
   3. In how many ways can the manager arrange all the TV sets if he is concerned about the order they were placed in?
3. Seven slips of paper with the digits to are placed in a large jar. After thoroughly mixing the slips of paper, two slips are picked without replacement.
   1. Explain the difference between and in terms of the digits selected.
   2. Describe a situation in which is the total number of outcomes.
   3. Describe a situation in which is the total number of outcomes.
   4. What is the relationship between and ?
4. If you know , and you also know the value of and , how could you find the value of ? Explain your answer.

**Example 1: Calculating Probabilities**

In a high school there are math teachers. The principal wants to form a committee by selecting three math teachers random. If Mr. H, Ms. B, and Ms. J are among the group of math teachers, what is the probability that all three of them will be on the committee?

Because every different committee of is equally likely,

The total number of possible committees is the number of ways that three math teachers can be chosen from math teachers, which is the number of combinations of math teachers taken at a time or . Mr. H, Ms. B, and Ms. J form one of these selections. The probability that the committee will consist of Mr. H, Ms. B, and Ms. J is .

Exercises 7–9

1. A high school is planning to put on the musical *West Side Story*. There are singers auditioning for the musical. The director is looking for two singers who could sing a good duet.
   1. What is the probability that Alicia and Juan are the two singers who are selected by the director? How did you get your answer?
   2. The director is also interested in the number of ways to choose a lead singer and a backup singer. What is the probability that Alicia is selected the lead singer and Juan is selected the backup singer? How did you get your answer?
2. For many computer tablets, the owner can set a -digit pass code to lock the device.
   1. How many different -digit pass codes are possible if the digits cannot be repeated? How did you get your answer?
   2. If the digits of a pass code are chosen at random and without replacement from the digits , , … , , what is the probability that the pass code is ? How did you get your answer?
   3. What is the probability that two people, who both chose a pass code by selecting digits at random and without replacement, both have a pass code of ? Explain your answer.
3. A chili recipe calls for ground beef, beans, green pepper, onion, chili powder, crushed tomatoes, salt, and pepper. You have lost the directions about the order in which to add the ingredients, so you decide to add them in a random order.
   1. How many different ways are there to add the ingredients? How did you get this answer?
   2. What is the probability that the first ingredient that you add is crushed tomatoes? How did you get your answer?
   3. What is the probability that the ingredients are added in the exact order listed above? How did you get your answer?

**Example 2: Probability and Combinations**

A math class consists of girls and boys. The teacher likes to have the students come to the board to demonstrate how to solve some of the math problems. During a lesson the teacher randomly selects of the students to show their work. What is the probability that all of the students selected are girls?

The number of ways to select girls from the girls is the number of combinations of from which is   
. The total number of groups of is .

The probability that all students are girls is

Exercises 10–11

1. There are nine golf balls numbered from to in a bag. Three balls are randomly selected without replacement to form a -digit number.
   1. How many -digit numbers can be formed? Explain your answer.
   2. How many -digit numbers start with the digit ? Explain how you got your answer.
   3. What is the probability that the -digit number formed is less than ? Explain your answer.
2. There are eleven seniors and five juniors who are sprinters on the high school track team. The coach must select four sprinters to run the -meter relay race.
   1. How many -sprinter relay teams can be formed from the group of sprinters?
   2. In how many ways can two seniors be chosen to be part of the relay team?
   3. In how many ways can two juniors be chosen to be part of the relay team?
   4. In how many ways can two seniors and two juniors be chosen to be part of the relay team?
   5. What is the probability that two seniors and two juniors will be chosen for the relay team?

Lesson Summary

* The number of permutations of things taken at a time is
* The number of combinations of items selected from a set of distinct items is
* Permutations and combinations can be used to calculate probabilities.

Problem Set

1. For each of the following, indicate whether it is a question that involves permutations, combinations, or neither, and then answer the question posed. Explain your reasoning.
   1. How many ways can a coach choose two co-captains from players in the basketball team?
   2. In how many ways can seven questions out of ten be chosen on an examination?
   3. Find the number of ways that women in the finals of the skateboard street competition can finish first, second, and third in the X Games final.
   4. A postal zip code contains five digits. How many different zip codes can be made with the digits –? Assume a digit can be repeated.
2. Four pieces of candy are drawn at random from a bag containing five orange pieces and seven brown pieces.
   1. How many different ways can four pieces be selected from the colored pieces?
   2. How many different ways can two orange pieces be selected from five orange pieces?
   3. How many different ways can two brown pieces be selected from seven brown pieces?
3. Consider the following:
   1. A game was advertised as having a probability of of winning. You know that the game involved five cards with a different digit on each card. Describe a possible game involving the cards that would have a probability of of winning.
   2. A second game involving the same five cards was advertised as having a winning probability of . Describe a possible game that would have a probability of or close to of winning
4. You have five people who are your friends on a certain social network. You are related to two of the people, but you do not recall who of the five people are your relatives. You are going to invite two of the five people to a special meeting. If you randomly select two of the five people to invite, explain how you would derive the probability of inviting your relatives to this meeting?
5. Charlotte is picking out her class ring. She can select from a ruby, an emerald, or an opal stone, and she can also select silver or gold for the metal.
   1. How many different combinations of one stone and one type of metal can she choose? Explain how you got your answer.
   2. If Charlotte selects a stone and a metal at random, what is the probability that she would select a ring with a ruby stone and gold metal?
6. In a lottery, three numbers are chosen from to . You win if the three numbers you pick match the three numbers selected by the lottery machine.
   1. What is the probability of winning this lottery if the numbers cannot be repeated?
   2. What is the probability of winning this lottery if the numbers can be repeated?
   3. What is the probability of winning this lottery if you must match the exact order that the lottery machine picked the numbers?
7. The store at your school wants to stock t-shirts that come in five sizes (small, medium, large, XL, XXL) and in two colors (orange and black).
   1. How many different type t-shirts will the store have to stock?
   2. At the next basketball game, the cheerleaders plan to have a t-shirt toss. If they have one t-shirt of each type in a box and select a shirt at random, what is the probability that the first randomly selected t-shirt is a large orange t-shirt?
8. There are balls in a bag numbered from to . Three balls are selected at random without replacement.
   1. How many different ways are there of selecting the three balls?
   2. What is the probability that one of the balls selected is the number ?
9. There are nine slips of paper in a bag numbered from to in a bag. Four slips are randomly selected without replacement to form a 4-digit number.
   1. How many -digit numbers can be formed?
   2. How many -digit numbers start with the digit ?
   3. What is the probability that the -digit number formed is less than ?
10. There are fourteen juniors and twenty-three seniors in the Service Club.  The club is to send four representatives to the State Conference.
    1. How many different ways are there to select a group of four students to attend the conference from the Service Club members?
    2. How many ways are there to select exactly two juniors?
    3. How many ways are there to select exactly two seniors?
    4. If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?
    5. What is the probability that two juniors and two seniors are selected to attend the conference?
11. A basketball team of players consists of guards, forwards, and centers. Coach decides to randomly select players to start the game. What is the probability of guards, forwards, and center starting the game?
12. A research study was conducted to estimate the number of white perch (a type of fish) in a Midwestern lake. perch were captured and tagged. After they were tagged, the perch were released back into the lake. A scientist involved in the research estimates there are perch in this lake. Several days after tagging and releasing the fish, the scientist caught perch of which were tagged. If this scientist’s estimate about the number of fish in the lake is correct, do you think it was likely to get perch out of with a tag? Explain your answer.