



Lesson 1: The General Multiplication Rule

Student Outcomes

- Students use the general multiplication rule to calculate the probability of the intersection of two events.
- Students interpret probabilities in context.

Lesson Notes

Grade 7 introduced students to the basic ideas of probability. This was extended in Grades 9 and 11 where students learned two-way tables and conditional probability. In this lesson, the multiplication rule is developed and applied in several different contexts.

The first example asks students to list all the possible outcomes for an experiment that consists of two steps. You may wish to remind students that a tree diagram is one method to list all the possibilities. Tree diagrams can be useful in students' development of probabilistic ideas. Tree diagrams were analyzed from an algebraic perspective in Algebra II, Module 1, Lesson 7. Exercise 1 is designed to introduce the fundamental counting principle as a method for finding the total number of possibilities rather than listing them all.

Grade 9 introduced students to the intersection of two events by using two-way tables. Students may be familiar with the idea of finding $P(A \text{ and } B)$ and with the concepts of independent and dependent events. This lesson extends these ideas to a general multiplication rule for calculating $P(A \text{ and } B)$, and there is further discussion of independent and dependent events.

Classwork

Example 1 (3 minutes): Independent Events

Many cereal companies placed prizes or toys in their cereal boxes to generate interest in buying their cereal. Students may recall their own interest in getting a prize or toy in a cereal box. You might want to show some of the prizes available by doing an Internet search on cereal box prizes.

Discuss Example 1 with students.

Example 1: Independent Events

Do you remember when breakfast cereal companies placed prizes in boxes of cereal? Possibly you recall that when a certain prize or toy was particularly special to children, it increased their interest in trying to get that toy. How many boxes of cereal would a customer have to buy to get that toy? Companies used this strategy to sell their cereal.

One of these companies put one of the following toys in its cereal boxes: a block (B), a toy watch (W), a toy ring (R), and a toy airplane (A). A machine that placed the toy in the box was programmed to select a toy by drawing a random number of 1 to 4. If a 1 was selected, the block (or B) was placed in the box; if a 2 was selected, a watch (or W) was placed in the box; if a 3 was selected, a ring (or R) was placed in the box; and if a 4 was selected, an airplane (or A) was placed in the box. When this promotion was launched, young children were especially interested in getting the toy airplane.



Exercises 1–8 (15 minutes)

Before students begin work on the Exercises, give them an opportunity to think, talk, write, and try to solve an overall question such as the following:

- What's the probability of finding (choose a prize) in two boxes in a row? Three? Explain how you determined your answer.

Give students time to struggle with this a little and present their answers before working through the questions below.

Allow 10 minutes for students to work through Exercises 1–4 in small groups (2 or 3 students). After they have a chance to answer the Exercises, discuss some of the explanations as a whole group.

Scaffolding:

Teachers may consider doing a physical demonstration to accompany this explanation with a cereal box and sample prizes (which could just be slips of paper with B, W, R, and A written on them).

Exercises 1–8

- If you bought one box of cereal, what is your estimate of the probability of getting the toy airplane? Explain how you got your answer.

The probability is $\frac{1}{4}$. I got this answer based on a sample space of {B, W, R, A}. Each outcome has the same chance of occurring. Therefore, the probability of getting the airplane is $\frac{1}{4}$.

- If you bought a second box of cereal, what is your estimate of the probability of getting the toy airplane in the second box? Explain how you got your answer.

The probability is again $\frac{1}{4}$. Since the machine that places a toy in the box picks a random number from 1 to 4, the probability that the second toy will be an airplane will again be $\frac{1}{4}$.

- If you bought two boxes of cereal, does your chance of getting at least one airplane increase or decrease? Explain your answer.

The probability of getting at least one airplane increases because the possible outcomes include the following an airplane in the first box but not the second box, an airplane in the second box but not the first box, and an airplane in both the first and second boxes.

- Do you think the probability of getting at least one airplane from two boxes is greater than 0.5? Again, explain your answer.

I think the probability is less than 0.5 as there are many more outcomes that I can describe that do not include the airplane. For example, you could get a watch and a block, a watch and a watch, a watch and ring, and so on.

Encourage each group to answer Exercise 5 without suggesting a specific strategy, as students could organize their lists in different ways. After students have formed their lists for Exercise 5, discuss the various strategies they used. If no group used a tree diagram, this would be a good time to review the design of a tree diagram for two events, which was developed in Grade 7. If students have not previously worked with a tree, this exercise provides an opportunity to present a tree as a way to organize the outcomes in a systemic list. A tree diagram provides a strategy to organize and analyze outcomes in other problems presented in this module.

5. List all of the possibilities of getting two toys from two boxes of cereals. (Hint: Think of the possible outcomes as ordered pairs. For example, BA would represent a block from the first box and an airplane from the second box.)

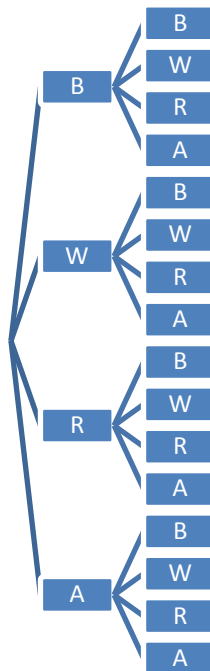
Consider the following ways students might create their lists using the notation:

B for block, W for watch, R for ring, and A for airplane.

The first letter represents the toy found in the first box, and the second letter represents the toy found in the second box. The first column represents getting the block in the first box, followed by each one of the other toys. The second column represents getting the watch in the first box, followed by each one of the other toys. The third column is developed with the ring in the first box, and the fourth column is developed with the airplane in the first box.

BB	WB	RB	AB
BW	WW	RW	AW
BR	WR	RR	AR
BA	WA	RA	AA

The following represents a tree diagram to form the lists:



Direct students to answer the following exercises using an organized list or the tree diagram.

6. Based on the list you created, what do you think is the probability of each of the following outcomes if two cereal boxes are purchased?
- One (and only one) airplane
 $\frac{6}{16}$ or $\frac{3}{8}$
 - At least one airplane
 $\frac{7}{16}$
 - No airplanes
 $\frac{9}{16}$
7. Consider the purchase of two cereal boxes.
- What is the probability of getting an airplane in the first cereal box? Explain your answer.
The probability is $\frac{1}{4}$ because there are 4 possible toys, and each is equally likely to be in the box.
 - What is the probability of getting an airplane in the second cereal box?
Again, the probability is $\frac{1}{4}$ because there are 4 possible toys, and each is equally likely to be in the box.
 - What is the probability of getting airplanes in both cereal boxes?
The probability is $\frac{1}{16}$ because there is 1 pair in the list (AA) out of the 16 that fits this description.

Point out that the event of getting an airplane in the first box purchased and then getting an airplane in the second box purchased is an example of what are called *independent events*. Students worked with independent and dependent events in Algebra II. Review that two events are independent of each other if knowing that one event has occurred does not change the probability that the second event occurs. Point out that because of the way toys are placed in the boxes, knowing the type of toy placed in the first cereal box does not tell us anything about what toy will be found in the second cereal box. Discuss the following summary with students:

The probability of A and B is denoted as $P(A \text{ and } B)$. This is also called the probability of A intersect B or $P(A \text{ "intersect" } B)$. If A and B are independent events, then the multiplication rule for independent events applies:

$P(A \text{ and } B)$ is the probability that Events A and B both occur and is the probability of the intersection of A and B . The probability of the intersection of Events A and B is sometimes also denoted by $P(A \cap B)$.

Multiplication Rule for Independent Events

If A and B are independent events, $P(A \text{ and } B) = P(A) \cdot P(B)$.

This rule generalizes to more than two independent events, for example:

$P(A \text{ and } B \text{ and } C)$ or $P(A \text{ intersect } B \text{ intersect } C) = P(A) \cdot P(B) \cdot P(C)$.

Students were introduced to intersection using Venn diagrams in earlier grades. Revisiting a Venn diagram might be helpful in reviewing the intersection of two events. Use Exercise 8 to informally assess student understanding so far.

8. Based on the multiplication rule for independent events, what is the probability of getting an airplane in both boxes? Explain your answer.

The probability would be $\frac{1}{16}$ as I would multiply the probability of getting an airplane in the first box ($\frac{1}{4}$) by the probability of getting an airplane in the second box, which is also $\frac{1}{4}$.

Scaffolding:

- In addition to using Venn diagrams, if students have trouble identifying the intersection of two events, point out real-world examples such as “intersecting streets” to connect students to the concept.
- For students above grade-level, pose the following: “What is the probability that you would *not* find an airplane in two boxes? Three? Four? Ten?” Explain how to determine these procedures.

Example 2 (2 minutes): Dependent Events

Example 2 moves to a discussion of dependent events. As students discuss this example, ask them how it differs from the first example.

If students are not familiar with the movie *Forrest Gump*, you may wish to show a short clip of the video where Forrest says, “Life is like a box of chocolates.” The main idea of this example is that as a piece of chocolate is chosen from a box, the piece is not replaced. Since the piece is not replaced, there is one less piece, so the number of choices for the second piece changes. The probability of getting a particular type of chocolate on the second selection is dependent on what type was chosen first.

Two events are dependent if knowing that one event occurring changes the probability that the other event occurs. This second probability is called a *conditional probability*. Students learned about conditional probabilities in several lessons in Algebra II. If this is their first time thinking about the conditional probabilities, use this example to indicate how the probability of the second selection is based on what the first selection was (or the *condition* of the first event).

Example 2: Dependent Events

Do you remember the famous line, “Life is like a box of chocolates,” from the movie *Forrest Gump*? When you take a piece of chocolate from a box, you never quite know what the chocolate will be filled with. Suppose a box of chocolates contains 15 identical-looking pieces. The 15 are filled in this manner: 3 caramel, 2 cherry cream, 2 coconut, 4 chocolate whip, and 4 fudge.

Exercises 9–14 (10 minutes)

Students should work in small groups (2 or 3 students per group). Allow about five minutes for the students to complete Exercises 9 and 10. When they have finished, discuss the answers. Then introduce the definition of dependent events and the multiplication rule for dependent events. Emphasize the symbol $P(A|B)$ and its meaning.

At the end of the lesson, the rule will be referred to as the *general multiplication rule* because it will work for any event, independent or dependent (given $P(B|A) = P(B)$ if A and B are independent).

Scaffolding:

For struggling students, present a simpler problem and scaffold. In a box of 8 crayons of 8 different colors (red, orange, yellow, green, blue, purple, brown, black), what is the probability that you pick a red crayon? What is the probability your neighbor will pick a yellow from the crayons left? Then go to a box of 16 crayons with 2 of each color and ask the same questions.

Exercises 9–14

9. If you randomly select one of the pieces of chocolate from the box, what is the probability that the piece will be filled with fudge?

$$\frac{4}{15} \approx 0.2667$$

10. If you randomly select a second piece of chocolate (after you have eaten the first one, which was filled with fudge), what is the probability that the piece will be filled with caramel?

$$\frac{3}{14} \approx 0.2143$$

Allow about five minutes for discussion of the multiplication rule for dependent events and for students to complete Exercises 11–14.

Before students begin the exercises, use the following as an opportunity to check for understanding at this phase:

- Ask students to restate the definition in their own words, either in writing or to a partner.
 - *Sample response: If two events are dependent, then I can find the probability of both the event A and B by multiplying the probability of event A by the probability of B given A .*

The events, *picking a fudge-filled piece on the first selection and picking a caramel-filled piece on the second selection*, are called dependent events.

Two events are dependent if knowing that one has occurred changes the probability that the other occurs.

Multiplication Rule for Dependent Events

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Recall from your previous work with probability in Algebra II that $P(B|A)$ is the conditional probability of event B given that event A occurred. If event A is *picking a fudge-filled piece on the first selection* and event B is *picking a caramel-filled piece on the second selection*, then $P(B|A)$ represents the probability of picking a caramel-filled piece second knowing that a fudge-filled piece was selected first.

11. If $A1$ is the event *picking a fudge-filled piece on the first selection* and $B2$ is the event *picking a caramel-filled piece on the second selection*, what does $P(A1 \text{ and } B2)$ represent? Find $P(A1 \text{ and } B2)$.

$P(A1 \text{ and } B2)$ represents the probability of picking a fudge-filled piece first and a caramel-filled piece second:

$$\frac{4}{15} \cdot \frac{3}{14} \approx 0.057$$

12. What does $P(B1 \text{ and } A2)$ represent? Calculate this probability.

$P(B1 \text{ and } A2)$ represents the probability of picking a caramel-filled piece first and a fudge-filled piece second:

$$\frac{3}{15} \cdot \frac{4}{14} \approx 0.057$$

13. If C represents selecting a coconut-filled piece of chocolate, what does $P(A1 \text{ and } C2)$ represent? Find this probability.

$P(A1 \text{ and } C2)$ represents the probability of picking a fudge-filled piece first and a coconut-filled piece second:

$$\frac{4}{15} \cdot \frac{2}{14} \approx 0.038$$

14. Find the probability that both the first and second pieces selected are filled with chocolate whip.

$$\frac{4}{15} \cdot \frac{3}{14} \approx 0.057$$

Exercises 15–17 (8 minutes)

Students should work in small groups (2 or 3 students per group). Allow them about five minutes to complete Exercises 15–17. When students have finished, discuss the answers. Students reason abstractly and quantitatively as these exercises provide practice in deciding if events are independent or dependent and practice in calculating probability. If students struggle, discuss that selections which occur with replacement are independent events, and selections which occur without replacement are dependent events.

Exercises 15–17

15. For each of the following, write the probability as the intersection of two events. Then, indicate whether the two events are independent or dependent, and calculate the probability of the intersection of the two events occurring.

- a. The probability of selecting a 6 from the first draw and a 7 on the second draw when two balls are selected without replacement from a container with 10 balls numbered 1 to 10.

Dependent

$$P(6 \text{ first and } 7 \text{ second}) = P(6 \text{ first}) \cdot P(7 \text{ second} | 6 \text{ first}) = \frac{1}{10} \cdot \frac{1}{9} = \frac{1}{90} \approx 0.011$$

- b. The probability of selecting a 6 on the first draw and a 7 on the second draw when two balls are selected with replacement from a container with 10 balls numbered 1 to 10.

Independent

$$P(6 \text{ first and } 7 \text{ second}) = P(6 \text{ first}) \cdot P(7 \text{ second}) = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} = 0.01$$

- c. The probability that two people selected at random in a shopping mall on a very busy Saturday both have a birthday in the month of June. Assume that all 365 birthdays are equally likely, and ignore the possibility of a February 29 leap-year birthday.

Independent

$$\begin{aligned} P(\text{first person June birthday and second person June birthday}) &= \\ P(\text{first person June birthday}) \cdot P(\text{second person June birthday}) &= \\ \frac{30}{365} \cdot \frac{30}{365} &= 0.0068 \end{aligned}$$

- d. The probability that two socks selected at random from a drawer containing 10 black socks and 6 white socks will both be black.

Dependent

$$\begin{aligned} P(\text{first sock black and second sock black}) &= \\ P(\text{first sock black}) \cdot P(\text{second sock black} | \text{first sock black}) &= \\ = \frac{10}{16} \cdot \frac{9}{15} &= \\ = 0.375 \end{aligned}$$

16. A gumball machine has gumballs of 4 different flavors: sour apple (*A*), grape (*G*), orange (*O*), and cherry (*C*). There are six gumballs of each flavor. When 50¢ is put into the machine, two random gumballs come out. The event *C1* means a cherry gumball came out first, the event *C2* means a cherry gumball came out second, the event *A1* means sour apple gumball came out first, and the event *G2* means a grape gumball came out second.

- a. What does $P(C2|C1)$ mean in this context?

The probability of the second gumball being cherry knowing the first gumball was cherry

- b. Find $P(C1 \text{ and } C2)$.

$$\frac{6}{24} \cdot \frac{5}{23} \approx 0.054$$

- c. Find $P(A1 \text{ and } G2)$.

$$\frac{6}{24} \cdot \frac{6}{23} \approx 0.0652$$

17. Below are the approximate percentages of the different blood types for people in the United States.

Type *O* 44%

Type *A* 42%

Type *B* 10%

Type *AB* 4%

Consider a group of 100 people with a distribution of blood types consistent with these percentages. If two people are randomly selected with replacement from this group, what is the probability that

- a. both people have type *O* blood?

$$0.44 \cdot 0.44 = 0.1936$$

- b. the first person has type A blood and the second person has type AB blood?

$$0.42 \cdot 0.04 = 0.0168$$

Closing (2 minutes)

- Ask students to summarize the key ideas of the lesson in writing or by talking to a neighbor. Use this as an opportunity to informally assess student understanding. The lesson summary provides some of the key ideas from the lesson.

Lesson Summary

- Two events are independent if knowing that one occurs does not change the probability that the other occurs.
- Two events are dependent if knowing that one occurs changes the probability that the other occurs.

- GENERAL MULTIPLICATION RULE:**

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

If A and B are independent events then $P(B|A) = P(B)$.

Exit Ticket (5 minutes)

Name _____

Date _____

Lesson 1: The General Multiplication Rule

Exit Ticket

Serena is in a math class of 20 students. Each day for a week (Monday to Friday), a student in Serena's class is randomly selected by the teacher to explain a homework problem. Once a student's name is selected, that student is not eligible to be selected again that week.

1.
 - a. What is the probability that Serena is selected on Monday?
 - b. What is the probability that Serena is selected on Tuesday given that she was not selected on Monday?
 - c. What is the probability that she will be selected on Friday given that she was not selected on any of the other days?
2. Suppose A represents Serena being selected, and B represents Dominic (another student in class) being selected. The event $A1$ means Serena was selected on Monday, and the event $B2$ means Dominic was selected on Tuesday. The event $B1$ means Dominic was selected on Monday, and the event $A2$ means Serena was selected on Tuesday.
 - a. Explain in words what $P(A1 \text{ and } B2)$ represents, and then calculate this probability.
 - b. Explain in words what $P(B1 \text{ and } A2)$ represents, and then calculate this probability.

Exit Ticket Sample Solutions

Serena is in a math class of 20 students. Each day for a week (Monday to Friday), a student in Serena's class is randomly selected by the teacher to explain a homework problem. Once a student's name is selected, that student is not eligible to be selected again that week.

1.

- a. What is the probability that Serena is selected on Monday?

$$\frac{1}{20} = 0.05$$

- b. What is the probability that Serena is selected on Tuesday given that she was not selected on Monday?

$$\frac{1}{19} \approx 0.0526$$

- c. What is the probability that she will be selected on Friday given that she was not selected on any of the other days?

$$\frac{1}{16} \approx 0.0625$$

2. Suppose A represents Serena being selected, and B represents Dominic (another student in class) being selected. The event $A1$ means Serena was selected on Monday, and the event $B2$ means Dominic was selected on Tuesday. The event $B1$ means Dominic was selected on Monday, and the event $A2$ means Serena was selected on Tuesday.

- a. Explain in words what $P(A1 \text{ and } B2)$ represents, and then calculate this probability.

It is the probability that Serena is selected on Monday and Dominic is selected on Tuesday.

$$P(A1 \text{ and } B2) = \frac{1}{20} \cdot \frac{1}{19} \approx 0.0026$$

- b. Explain in words what $P(B1 \text{ and } A2)$ represents, and then calculate this probability.

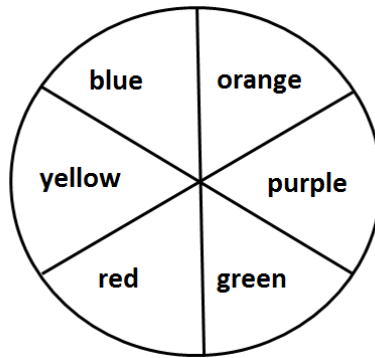
It is the probability that Dominic is selected on Monday and Serena is selected on Tuesday.

$$P(B1 \text{ and } A2) = \frac{1}{20} \cdot \frac{1}{19} \approx 0.0026$$

Problem Set Sample Solutions

Use this space to describe any specific details about the problem set for teacher reference.

1. In a game using the spinner below, a participant spins the spinner twice. If the spinner lands on red both times, the participant is a winner.



- a. The event *participant is a winner* can be thought of as the intersection of two events. List the two events.

First spin lands on red and second spin lands on red.

- b. Are the two events independent? Explain.

Independent—knowing the first spin landed on red does not change the probability of the second spin landing on red.

- c. Find the probability that a participant wins the game.

$$\frac{1}{6} \cdot \frac{1}{6} \approx 0.0278$$

2. The overall probability of winning a prize in a weekly lottery is $\frac{1}{32}$. What is the probability of winning a prize in this lottery three weeks in a row?

$$\frac{1}{32} \cdot \frac{1}{32} \cdot \frac{1}{32} \approx 0.00003$$

3. A Gallup poll reported that 28% of adults (age 18 and older) eat at a fast food restaurant about once a week. Find the probability that two randomly selected adults would both say they eat at a fast food restaurant about once a week.

$$0.28 \cdot 0.28 = 0.0784$$

4. In the game *Scrabble*, there are a total of 100 tiles. Of the 100 tiles, 42 tiles have the vowels A, E, I, O, and U printed on them, 56 tiles have the consonants printed on them, and 2 tiles are left blank.

- a. If tiles are selected at random, what is the probability that the first tile drawn from the pile of 100 tiles is a vowel?

$$\frac{42}{100} = 0.42$$

- b. If tiles drawn are not replaced, what is the probability that the first two tiles selected are both vowels?

$$\frac{42}{100} \cdot \frac{41}{99} \approx 0.174$$

- c. Event *A* is *drawing a vowel*, event *B* is *drawing a consonant*, and event *C* is *drawing a blank tile*. *A1* means a vowel is drawn on the first selection, *B2* means a consonant is drawn on the second selection, and *C2* means a blank tile is drawn on the second selection. Tiles are selected at random and without replacement.

i. Find $P(A1 \text{ and } B2)$ $= \frac{42}{100} \cdot \frac{56}{99} \approx 0.238$

ii. Find $P(A1 \text{ and } C2)$ $= \frac{42}{100} \cdot \frac{2}{99} \approx 0.008$

iii. Find $P(B1 \text{ and } C2)$ $= \frac{56}{100} \cdot \frac{2}{99} \approx 0.011$

5. To prevent a flooded basement, a homeowner has installed two special pumps that work automatically and independently to pump water if the water level gets too high. One pump is rather old and does not work 28% of the time, and the second pump is newer and does not work 9% of the time. Find the probability that both pumps will fail to work at the same time.

$$0.28 \cdot 0.09 \approx 0.025$$

6. According to a recent survey, approximately 77% of Americans get to work by driving alone. Other methods for getting to work are listed in the table below.

Method of getting to work	Percent of Americans using this method
Taxi	0.1%
Motorcycle	0.2%
Bicycle	0.4%
Walk	2.5%
Public Transportation	4.7%
Car Pool	10.7%
Drive Alone	77%
Work at Home	3.7%
Other	0.7%

- a. What is the probability that a randomly selected worker drives to work alone?

$$0.77$$

- b. What is the probability that two workers selected at random with replacement both drive to work alone?

$$\text{Assume independent } 0.77 \cdot 0.77 \approx 0.593$$

7. A bag of M&Ms contains the following distribution of colors:

9 blue
6 orange
5 brown
5 green
4 red
3 yellow

Three M&Ms are randomly selected without replacement. Find the probabilities of the following events.

- a. All three are blue.

$$\frac{9}{32} \cdot \frac{8}{31} \cdot \frac{7}{30} \approx 0.017$$

- b. The first one selected is blue, the second one selected is orange, and the third one selected is red.

$$\frac{9}{32} \cdot \frac{6}{31} \cdot \frac{4}{30} \approx 0.007$$

- c. The first two selected are red, and the third one selected is yellow.

$$\frac{4}{32} \cdot \frac{3}{31} \cdot \frac{3}{30} \approx 0.001$$

8. Suppose in a certain breed of dog, the color of fur can either be tan or black. Eighty-five percent of the time, a puppy will be born with tan fur, while 15% of the time, the puppy will have black fur. Suppose in a future litter, six puppies will be born.

- a. Are the events *having tan fur* and *having black fur* independent? Explain.

Yes, knowing the color of fur for one puppy doesn't affect the probability of fur color for another puppy.

- b. What is the probability that one puppy in the litter will have black fur and another puppy will have tan fur?

$$0.15 \cdot 0.85 = 0.1275$$

- c. What is the probability that all six puppies will have tan fur?

$$0.85 \cdot 0.85 \cdot 0.85 \cdot 0.85 \cdot 0.85 \cdot 0.85 \approx 0.377$$

- d. Is it likely for three out of the six puppies to be born with black fur? Justify mathematically.

No, the probability of three puppies being born with black fur is $0.15 \cdot 0.15 \cdot 0.15 = 0.003375$. This is not likely to happen.

9. Suppose that in the litter of six puppies from Exercise 8, five puppies are born with tan fur, and one puppy is born with black fur.

- a. You randomly pick up one puppy. What is the probability that puppy will have black fur?

$$\frac{1}{6} \approx 0.167$$

- b. You randomly pick up one puppy, put it down, and randomly pick up a puppy again. What is the probability that both puppies will have black fur?

$$\frac{1}{6} \cdot \frac{1}{6} \approx 0.028$$

- c. You randomly pick up two puppies, one in each hand. What is the probability that both puppies will have black fur?

0; this outcome can never happen since there is only one black puppy.

- d. You randomly pick up two puppies, one in each hand. What is the probability that both puppies will have tan fur?

$$\frac{5}{6} \cdot \frac{4}{5} \approx 0.667$$