

Lesson 15: Transforming Rational Functions

Classwork

Exploratory Challenge/Exercises 1–5

1. Sketch the general shape of the graph of the function $f(x) = \frac{1}{x^n}$ for $n > 0$ when n is an odd number.
2. Sketch the general shape of the graph of the function $f(x) = \frac{1}{x^n}$ for $n > 0$ when n is an even number.

3. Sketch the graph of the function $f(x) = \frac{1}{x}$. Then, use the graph of f to sketch each transformation of f showing the vertical and horizontal asymptotes.

a. $g(x) = \frac{1}{x-2}$

b. $h(x) = -\frac{1}{x} + 3$

c. $k(x) = \frac{2}{x+3} - 5$

4. Use your results from Exercise 3 to make some general statements about graphs of functions in the form $f(x) = a + \frac{b}{x-c}$. Describe the effect that changing each parameter a , b , and c has on the graph of f .

5. Sketch the graph of the function $f(x) = \frac{1}{x^2}$. Then, use the graph of f to sketch each transformation of f showing the vertical and horizontal asymptotes.

a. $g(x) = -\frac{3}{(x+1)^2}$

b. $h(x) = \frac{1}{(x-1)^2} + 4$

Example 1

Graph the function $f(x) = \frac{x+2}{x-3}$ using transformations of the graph of $y = \frac{1}{x}$.

Exercises 6–13

Sketch each function by using transformations of the graph of $y = \frac{1}{x}$ or the graph of $y = \frac{1}{x^2}$. Explain the transformations that are evident in each example.

6. $f(x) = \frac{x-7}{x-5}$

7. $f(x) = \frac{2x+6}{x+1}$

8. $f(x) = \frac{2x^2-1}{x^2}$

9. $f(x) = \frac{1+4x^3}{x^3}$

10. $f(x) = \frac{x^2 - 2x + 3}{(x-1)^2}$

11. $f(x) = \frac{2x^2 + 12x + 13}{(x+3)^2}$

12. $f(x) = \frac{x+4}{x^2-16}$

13. $f(x) = \frac{x}{x^3-4x^2+4x}$

Problem Set

- Write each function so that it appears to be a transformation of $y = \frac{1}{x^n}$. Then, explain how the graph of each function relates to the graph of $y = \frac{1}{x^n}$.
 - $y = \frac{5x-8}{x+2}$
 - $y = \frac{2x^3-4}{x^3}$
 - $y = \frac{x^2-4x+8}{(x-2)^2}$
 - $y = \frac{3x-12}{x^2-16}$
 - $y = \frac{2x^2+16x+25}{x^2+8x+16}$
- For each function in Problem 1, state how the horizontal and vertical asymptotes are affected from the original graph of $y = \frac{1}{x^n}$.
- Sketch a picture of the graph of each function in Problem 1.
- What are some indicators that a rational function can be expressed as a transformation of $y = \frac{1}{x^n}$ or not?
- Write an equation for a function whose graph is a transformation of the graph $y = \frac{1}{x}$. The graph has been shifted right 2 units, stretched vertically by a factor of 2, and been shifted down 3 units.