Lesson 15: Transforming Rational Functions

Classwork

Exploratory Challenge/Exercises 1–5

1. Sketch the general shape of the graph of the function $f\left(x\right)=\frac{1}{x^{n}}$ for $n>0$ when $n$ is an odd number.
2. Sketch the general shape of the graph of the function $f\left(x\right)=\frac{1}{x^{n}}$ for $n>0$ when $n$ is an even number.
3. Sketch the graph of the function $f\left(x\right)=\frac{1}{x}$. Then, use the graph of $f$ to sketch each transformation of $f$ showing the vertical and horizontal asymptotes.
	1. $g(x)=\frac{1}{x-2}$
	2. $h\left(x\right)=-\frac{1}{x}+3$
	3. $k\left(x\right)=\frac{2}{x+3}-5$
4. Use your results from Exercise 3 to make some general statements about graphs of functions in the form
$f\left(x\right)=a+\frac{b}{x-c}$. Describe the effect that changing each parameter $a, b, and c$ has on the graph of $f.$
5. Sketch the graph of the function $f\left(x\right)=\frac{1}{x^{2}}$. Then, use the graph of $f$ to sketch each transformation of $f$ showing the vertical and horizontal asymptotes.
	1. $g\left(x\right)=-\frac{3}{\left(x+1\right)^{2}}$
	2. $h\left(x\right)=\frac{1}{\left(x-1\right)^{2}}+4$

**Example 1**

Graph the function $f\left(x\right)=\frac{x+2}{x-3}$ using transformations of the graph of $y=\frac{1}{x}.$

Exercises 6–13

Sketch each function by using transformations of the graph of $y=\frac{1}{x}$ or the graph of $y=\frac{1}{x^{2}}$. Explain the transformations that are evident in each example.

1. $f(x)=\frac{x-7}{x-5}$
2. $f(x)=\frac{2x+6}{x+1}$
3. $f(x)=\frac{2x^{2}-1}{x^{2}}$
4. $f(x)=\frac{1+4x^{3}}{x^{3}}$
5. $f(x)=\frac{x^{2}-2x+3}{\left(x-1\right)^{2}}$
6. $f(x)=\frac{2x^{2}+12x+13}{\left(x+3\right)^{2}}$
7. $f(x)=\frac{x+4}{x^{2}-16}$
8. $f(x)=\frac{x}{x^{3}-4x^{2}+4x}$

Problem Set

1. Write each function so that it appears to be a transformation of $y= \frac{1}{x^{n}}$. Then, explain how the graph of each function relates to the graph of $y=\frac{1}{x^{n}}$.
	1. $y=\frac{5x-8}{x+2}$
	2. $y=\frac{2x^{3}-4}{x^{3}}$
	3. $y=\frac{x^{2}-4x+8}{\left(x-2\right)^{2}}$
	4. $y=\frac{3x-12}{x^{2}-16}$
	5. $y=\frac{2x^{2}+16x+25}{x^{2}+8x+16}$
2. For each function in Problem 1, state how the horizontal and vertical asymptotes are affected from the original graph of $y=\frac{1}{x^{n}}$.
3. Sketch a picture of the graph of each function in Problem 1.
4. What are some indicators that a rational function can be expressed as a transformation of $y=\frac{1}{x^{n}} $or not?
5. Write an equation for a function whose graph is a transformation of the graph $y= \frac{1}{x}$. The graph has been shifted right 2 units, stretched vertically by a factor of 2, and been shifted down 3 units.