# Lesson 12: End Behavior of Rational Functions

## Classwork

### Opening Exercise

Analyze the end behavior of each function below. Then choose one of the functions and explain how you determined the end behavior.

a. $f(x) = x^4$

b. $g(x) = -x^4$

c. $h(x) = x^3$

d. $k(x) = -x^3$

**Exercises**

Determine the end behavior of each rational function below.

1. $f(x) = \frac{7x^5-3x+1}{4x^3+2}$

2. $f(x) = \frac{7x^3-3x+1}{4x^3+2}$

3. $f(x) = \frac{7x^3+2}{4x^5-3x+1}$

## Problem Set

1. Analyze the end behavior of both functions.
   a. $f(x) = x,\ g(x) = \frac{1}{x}$.
   b. $f(x) = x^3,\ g(x) = \frac{1}{x^3}$.
   c. $f(x) = x^2,\ g(x) = \frac{1}{x^2}$.
   d. $f(x) = x^4, g(x) = \frac{1}{x^4}$.
   e. $f(x) = x - 1, g(x) = \frac{1}{x-1}$.
   f. $f(x) = x + 2, g(x) = \frac{1}{x+2}$.
   g. $f(x) = x^2 - 4, g(x) = \frac{1}{x^2-4}$.

2. For the following functions, determine the end behavior. Confirm your answer with a table of values.
   a. $f(x) = \frac{3x-6}{x+2}$.
   b. $f(x) = \frac{5x+1}{x^2-x-6}$.
   c. $f(x) = \frac{x^3-8}{x^2-4}$.
   d. $f(x) = \frac{x^3-1}{x^4-1}$.
   e. $f(x) = \frac{(2x+1)^3}{(x^2-x)^2}$.

3. For the following functions, determine the end behavior.
   a. $f(x) = \frac{5x^6-3x^3+x-2}{5x^4-3x^3+x-2}$.
   b. $f(x) = \frac{5x^4-3x^3+x-2}{5x^6-3x^3+x-2}$.
   c. $f(x) = \frac{5x^4-3x^3+x-2}{5x^4-3x^3+x-2}$.
   d. $f(x) = \frac{\sqrt{2}x^2+x+1}{3x+1}$.
   e. $f(x) = \frac{4x^2-3x-7}{2x^3+x-2}$.

4. Determine the end behavior of each function.
   a. $f(x) = \frac{\sin x}{x}$.
   b. $f(x) = \frac{\cos x}{x}$.
   c. $f(x) = \frac{2^x}{x}$.

d. $f(x) = \frac{x}{2^x}$.

e. $f(x) = \frac{4}{1+e^{-x}}$.

f. $f(x) = \frac{10}{1+e^{-x}}$.

5. Consider the functions $f(x) = x!$ and $g(x) = x^5$ for natural numbers $x$.

   a. What are the values of $f(x)$ and $g(x)$ for $x = 5,10,15,20,25$?

   b. What is the end behavior of $f(x)$ as $x \to \infty$.

   c. What is the end behavior of $g(x)$ as $x \to \infty$.

   d. Make an argument for the end behavior of $\frac{f(x)}{g(x)}$ as $x \to \infty$.

   e. Make an argument for the end behavior of $\frac{g(x)}{f(x)}$ as $x \to \infty$.

6. Determine the end behavior of the functions.

   a. $f(x) = \frac{x}{x^2}$, $g(x) = \frac{1}{x}$.

   b. $f(x) = \frac{x+1}{x^2-1}$, $g(x) = \frac{1}{x-1}$.

   c. $f(x) = \frac{x-2}{x^2-x-2}$, $g(x) = \frac{1}{x+1}$.

   d. $f(x) = \frac{x^3-1}{x-1}$, $g(x) = x^2 + x + 1$.

7. Use a graphing utility to graph the following functions $f(x)$ and $g(x)$. Explain why they have the same graphs. Determine the end behavior of the functions and whether the graphs have any horizontal asymptotes.

   a. $f(x) = \frac{x+1}{x-1}$, $g(x) = 1 + \frac{2}{x-1}$.

   b. $f(x) = \frac{-2x+1}{x+1}$, $g(x) = \frac{3}{x+1} - 2$.