

Lesson 11: Rational Functions

Classwork

Opening Exercise

Factor each expression completely:

a. $9x^4 - 16x^2$

b. $2x^3 + 5x^2 - 8x - 20$

c. $x^3 + 3x^2 + 3x + 1$

d. $8x^3 - 1$

Example 1

Reduce the expression $\frac{x^2-5x+6}{x-3}$ to lowest terms and identify the value(s) of x that must be excluded to avoid division by zero.

Exercise 1

1. Reduce each rational expression to lowest terms, specifying the values of x that must be excluded to avoid division by zero.

a. $\frac{x^2-6x+5}{x^2-3x-10}$

b. $\frac{x^3+3x^2+3x+1}{x^3+2x^2+x}$

c. $\frac{x^2-16}{x^2+2x-8}$

d. $\frac{x^2-3x-10}{x^3+6x^2+12x+8}$

e. $\frac{x^3+1}{x^2+1}$

Example 2

Let $f(x) = \frac{2x^4+6x^3+6x^2+2x}{3x^2+3x}$. Reduce the rational expression $\frac{2x^4+6x^3+6x^2+2x}{3x^2+3x}$ to lowest terms and use the reduced form to express the rule of f . Be sure to indicate any restrictions on the domain.

Exercise 2

2. Determine the domain of each rational function and express the rule for each function in an equivalent form in lowest terms.

a. $f(x) = \frac{(x+2)^2(x-3)(x+1)}{(x+2)(x+1)}$

b. $f(x) = \frac{x^2 - 6x + 9}{x - 3}$

c. $f(x) = \frac{3x^3 - 75x}{x^3 + 15x^2 + 75x + 125}$

Problem Set

- For each pair of functions f and g , find the domain of f and the domain of g . Indicate whether f and g are the same function.
 - $f(x) = \frac{x^2}{x}$, $g(x) = x$
 - $f(x) = \frac{x}{x}$, $g(x) = 1$
 - $f(x) = \frac{2x^2+6x+8}{2}$, $g(x) = x^2 + 6x + 8$
 - $f(x) = \frac{x^2+3x+2}{x+2}$, $g(x) = x + 1$
 - $f(x) = \frac{x+2}{x^2+3x+2}$, $g(x) = \frac{1}{x+1}$
 - $f(x) = \frac{x^4-1}{x^2-1}$, $g(x) = x^2 + 1$
 - $f(x) = \frac{x^4-1}{x^2+1}$, $g(x) = x^2 - 1$
 - $f(x) = \frac{x^4-x}{x^2+x}$, $g(x) = \frac{x^3-1}{x+1}$
 - $f(x) = \frac{x^4+x^3+x^2}{x^2+x+1}$, $g(x) = x^2$
- Determine the domain of each rational function and express the rule for each function in an equivalent form in lowest terms.
 - $f(x) = \frac{x^4}{x^2}$
 - $f(x) = \frac{3x+3}{15x-6}$
 - $f(x) = \frac{x^2-x-2}{x^2+x}$
 - $f(x) = \frac{8x^2+2x-15}{4x^2-4x-15}$
 - $f(x) = \frac{2x^3-3x^2-2x+3}{x^3-x}$
 - $f(x) = \frac{3x^3+x^2+3x+1}{x^3+x}$
- For each pair of functions below, calculate $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$ and $\frac{f(x)}{g(x)}$. Indicate restrictions on the domain of the resulting functions.
 - $f(x) = \frac{2}{x}$, $g(x) = \frac{x}{x+2}$
 - $f(x) = \frac{3}{x+1}$, $g(x) = \frac{x}{x^3+1}$