Lesson 11: Rational Functions

Classwork

Opening Exercise

Factor each expression completely:

* 1. $9x^{4}-16x^{2}$
	2. $2x^{3}+5x^{2}-8x-20$
	3. $x^{3}+3x^{2}+3x+1$
	4. $8x^{3}-1$

**Example 1**

Reduce the expression $\frac{x^{2}-5x+6}{x-3 }$ to lowest terms and identify the value(s) of $x$ that must be excluded to avoid division by zero.

Exercise 1

1. Reduce each rational expression to lowest terms, specifying the values of $x$ that must be excluded to avoid division by zero.
	1. $\frac{x^{2}-6x+5}{x^{2}-3x-10}$
	2. $\frac{x^{3}+3x^{2}+3x+1}{x^{3}+2x^{2}+x}$
	3. $\frac{x^{2}-16}{x^{2}+2x-8}$
	4. $\frac{x^{2}-3x-10}{x^{3}+6x^{2}+12x+8}$
	5. $\frac{x^{3}+1}{x^{2}+1}$

**Example 2**

Let $f(x)=\frac{2x^{4}+6x^{3}+6x^{2}+2x}{3x^{2}+3x}$. Reduce the rational expression $\frac{2x^{4}+6x^{3}+6x^{2}+2x}{3x^{2}+3x} $to lowest terms and use the reduced form to express the rule of $f$. Be sure to indicate any restrictions on the domain.

Exercise 2

1. Determine the domain of each rational function and express the rule for each function in an equivalent form in lowest terms.
	1. $f(x)=\frac{\left(x+2\right)^{2}\left(x-3\right)\left(x+1\right)}{\left(x+2\right)\left(x+1\right)} $
	2. $f(x)=\frac{x^{2}-6x+9}{x-3}$
	3. $f(x)=\frac{3x^{3}-75x}{x^{3}+15x^{2}+75x+125}$

Problem Set

1. For each pair of functions $f$ and $g$, find the domain of $f$ and the domain of $g$. Indicate whether $f$ and $g$ are the same function.
	1. $f\left(x\right)=\frac{x^{2}}{x}, g\left(x\right)=x$
	2. $f\left(x\right)=\frac{x}{x}, g\left(x\right)=1$
	3. $f\left(x\right)=\frac{2x^{2}+6x+8}{2}, g\left(x\right)=x^{2}+6x+8$
	4. $f\left(x\right)=\frac{x^{2}+3x+2}{x+2}, g\left(x\right)=x+1$
	5. $f\left(x\right)=\frac{x+2}{x^{2}+3x+2}, $ $g\left(x\right)=\frac{1}{x+1}$
	6. $f\left(x\right)=\frac{x^{4}-1}{x^{2}-1}, g\left(x\right)=x^{2}+1$
	7. $f\left(x\right)=\frac{x^{4}-1}{x^{2}+1}, g\left(x\right)=x^{2}-1$
	8. $f\left(x\right)=\frac{x^{4}-x}{x^{2}+x} , $ $g\left(x\right)=\frac{x^{3}-1}{x+1}$
	9. $f\left(x\right)=\frac{x^{4}+x^{3}+x^{2}}{x^{2}+x+1}, $ $g\left(x\right)=x^{2}$
2. Determine the domain of each rational function and express the rule for each function in an equivalent form in lowest terms.
	1. $f\left(x\right)=\frac{x^{4}}{x^{2}} $
	2. $f\left(x\right)=\frac{3x+3}{15x-6}$
	3. $f\left(x\right)=\frac{x^{2}-x-2}{x^{2}+x}$
	4. $f\left(x\right)=\frac{8x^{2}+2x-15}{4x^{2}-4x-15}$
	5. $f\left(x\right)=\frac{2x^{3}-3x^{2}-2x+3}{x^{3}-x}$
	6. $f\left(x\right)=\frac{3x^{3}+x^{2}+3x+1}{x^{3}+x}$
3. For each pair of functions below, calculate $f\left(x\right)+g\left(x\right), f\left(x\right)-g\left(x\right), f\left(x\right)⋅g(x)$ and $\frac{f\left(x\right)}{g\left(x\right)}$. Indicate restrictions on the domain of the resulting functions.
	1. $f\left(x\right)=\frac{2}{x}, g\left(x\right)=\frac{x}{x+2}$
	2. $f\left(x\right)=\frac{3}{x+1}, g\left(x\right)=\frac{x}{x^{3}+1}$