Lesson 10: The Structure of Rational Expressions

Student Outcomes

- Students understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.
- Students add, subtract, multiply, and divide rational expressions.

Lesson Notes

In Algebra II, students performed operations on rational expressions. They learned that the process of combining rational expressions is analogous to that of combining rational numbers. While this lesson lets students review these skills, the focus here is on understanding that rational expressions form a system analogous to the rational numbers. In particular, students use the properties of integers to establish closure for the set of rational numbers, then they use properties of polynomials to establish closure for the set of rational expressions.

Note: Students are often directed to simplify rational expressions, which may require them to add, subtract, multiply, or divide two rational expressions and to reduce the resulting expression by dividing out common factors. The term *simplify* can prove problematic because it is not always clear whether the rational expression that results from the procedures above is simpler than the original expression. The goal is for students to write the rational expression so that there is a single polynomial denominator.

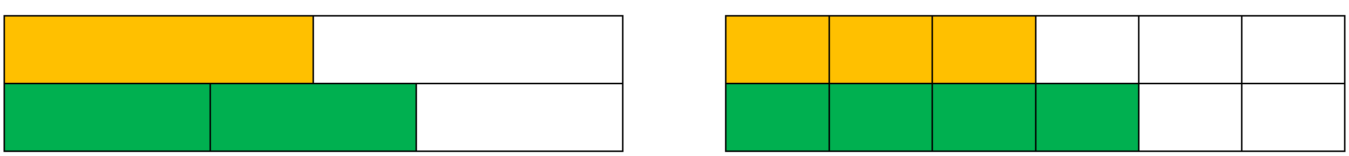
Classwork

Opening Exercise (6 minutes)

This lesson reviews what you learned in Algebra II about how to add, subtract, multiply, and divide rational expressions. Then, we establish a connection between the properties of rational expressions and those of rational numbers. In this set of exercises, students perform addition and subtraction: first with rational numbers, then with rational expressions.

The bar model below for $\frac{1}{2} + \frac{2}{3}$ can be presented to students as scaffolding if they need a reminder on how to add fractions.

The following representation shows that $\frac{1}{2} + \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{2}{2} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$.



Opening Exercise

1. **Add the fractions: $\frac{3}{5}+\frac{2}{7}$.**

$$\frac{3}{5}+\frac{2}{7}=\frac{3}{5}\cdot\frac{7}{7}+\frac{2}{7}\cdot\frac{5}{5}=\frac{21}{35}+\frac{10}{35}=\frac{31}{35}$$

2. **Subtract the fractions: $\frac{5}{2}-\frac{4}{3}$.**

$$\frac{5}{2}-\frac{4}{3}=\frac{5}{2}\cdot\frac{3}{3}-\frac{4}{3}\cdot\frac{2}{2}=\frac{15}{6}-\frac{8}{6}=\frac{7}{6}$$

3. **Add the expressions: $\frac{3}{x}+\frac{x}{5}$.**

$$\frac{3}{x}+\frac{x}{5}=\frac{3}{x}\cdot\frac{5}{5}+\frac{x}{5}\cdot\frac{x}{x}=\frac{15}{5x}+\frac{x^2}{5x}=\frac{15+x^2}{5x}$$

4. **Subtract the expressions: $\frac{x}{x+2}-\frac{3}{x+1}$.**

$$\frac{x}{x+2}-\frac{3}{x+1}=\frac{x}{x+2}\cdot\frac{x+1}{x+1}-\frac{3}{x+1}\cdot\frac{x+2}{x+2}=\frac{x^2+x}{(x+2)(x+1)}-\frac{3x+6}{(x+1)(x+2)}=\frac{(x^2+x)-(3x+6)}{(x+2)(x+1)}$$

Scaffolding:

Give cues to students as necessary using questions such as these:

- Do the fractions have a common denominator?
- What operations could you perform to get a common denominator?
- Demonstrate fraction addition using a bar model if necessary.

Discussion (5 minutes)

This discussion should lead students to consider the idea of closure and the connections between the structure of operations performed with rational numbers to those of operations performed with rational expressions.

MP.7

- How are Exercises 3 and 4 similar to Exercises 1 and 2? How are they different?
  - *In both cases, we need a common denominator in order to combine the expressions to form a single entity.*
  - *In the case of Exercises 1 and 2, the results are numbers, but in Exercises 3 and 4, the results are expressions that contain a variable.*
- In Exercise 1, you found that $\frac{3}{5}+\frac{2}{7}=\frac{31}{35}$. In Exercise 2, you found that $\frac{5}{2}-\frac{4}{3}=\frac{7}{6}$. What do these exercises illustrate about the sum and difference of rational numbers?
  - *The sum or difference of two rational numbers is a rational number.*
- Let's review why this is true. How do we define a rational number?
  - *It is the ratio of two integers, where the denominator does not equal zero.*

MP.3

- How can we reason why the sum or difference of two rational numbers is rational?
  - *Finding the numerator of the sum or difference of rational numbers requires us to add, subtract, and/or multiply integers. We find the denominator by multiplying integers. The product of two integers is an integer; likewise the sum and difference of two integers is an integer. Therefore, the numerators and denominators are both integers, which means that the sum or difference is a rational number.*
  - *The denominator is the product of two non-zero integers, so the product cannot be zero.*

- What is the mathematical word for this property, and where have we used it before?
  - *Closure: the students may recall discussing closure with respect to integers, rational numbers, and polynomials.*

Exercises 1–2 (8 minutes)

In this exercise, students use the technique shown above to construct an argument. Provide cues as needed to help them develop this argument. Encourage students to work together. Let them work for several minutes, and then select a student to present his or her argument to the class.

Exercises

1. Construct an argument that shows that the set of rational numbers is closed under addition. That is, if x and y are rational numbers and $w = x + y$, prove that w must also be a rational number.

Since x and y are rational numbers, there are four integers a, b, c, and d with $x = \frac{a}{b}$ and $y = \frac{c}{d}$, and neither b nor d is 0.

Now we need to check to see if w is a rational number:

$$w = x + y = \frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b} = \frac{ad + cb}{bd}$$

The numerator is formed by multiplying and adding integers, so it must be an integer. Similarly, the denominator must be an integer. Lastly, bd cannot be 0 since neither b nor d is 0. This proves that w is a rational number.

2. How could you modify your argument to show that the set of rational numbers is also closed under subtraction? Discuss your response with another student.

This time we start with $\frac{a}{b} - \frac{c}{d}$ and end up with $\frac{ad-cb}{bd}$. We just notice that subtracting two integers yields an integer, and then apply the same reasoning as before.

Discussion (7 minutes)

- Now that we've shown that the set of rational numbers is closed under addition, let's extend our thinking from the realm of numbers to the realm of algebra: Is the set of rational expressions also closed under addition? To help answer this question, let's return to the Opening Exercise.
- In the opening, you showed that $\frac{x}{x+2} - \frac{3}{x+1} = \frac{(x^2+x)-(3x+6)}{(x+2)(x+1)}$. Is this result a rational expression? We'll need to recall some information about what a rational expression is.

> All rational expressions can be put into the form $\frac{P}{Q}$ where P and Q are polynomial expressions and Q is not the zero polynomial. Rational expressions do not necessarily start out in this form, but all can be rewritten in it.

- Now let's examine the expression $\frac{(x^2+x)-(3x+6)}{(x+2)(x+1)}$. Does this expression meet the above requirement?
  - *Yes, the numerator involves subtracting two polynomials, and the denominator involves multiplying two polynomials, so the quotient is a rational expression.*

- This analysis should give you some idea of what happens in the general case. Our work with rational numbers hinged on our understanding of *integers;* our work with rational expressions will hinge on *polynomials*.
- We can make an argument for the closure of rational expressions under addition that closely parallels the argument we made about rational numbers. Work together with a partner to develop an argument to this end.
  - *If x and y be rational expressions, then there are polynomials a, b, c, and d so that $x = \frac{a}{b}$ and $y = \frac{c}{d}$, and neither b nor d is the zero polynomial.*
  - *The sum of x and y is $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.*
  - *The terms ad and bc are polynomials because they are products of polynomials, and polynomials are closed under multiplication.*
  - *The numerator $ad + bc$ is a polynomial because it is the sum of polynomials ad and bc, and polynomials are closed under addition.*
  - *The denominator bd is a polynomial because it is the product of polynomials b and d, and polynomials are closed under multiplication.*
  - *The sum is a rational number because the numerator and denominator are both polynomials.*
- In the case of integers, we know that bd cannot be zero unless either b or d is zero. Similarly, it can be shown that bd cannot be the zero polynomial unless b or d is the zero polynomial. So the expression $\frac{ad+bc}{bd}$ is a bona fide rational expression after all! We could make a similar argument to show that the set of rational expressions is closed under subtraction, also.
- Can you summarize the discussion so far? Try to convey the main point of the lesson to another student in one or two sentences.
  - *We showed that the set of rational numbers is closed under addition and subtraction, and then we showed that the set of rational expressions is closed under addition and subtraction, too.*
- Let's summarize the logic of the lesson as well: How did we establish closure for the set of rational numbers? How did we establish closure for the set of rational expressions? Make your answers as concise as possible.
  - *We established closure for the set of rational numbers by using closure properties for the set of integers; then we established closure for the set of rational expressions by using closure properties for the set of polynomials.*
- Now that we have studied the structure of addition and subtraction, let's turn our attention to multiplication and division.

Exercises 3–6 (4 minutes)

These exercises review how to multiply and divide fractions. Students then multiply and divide rational expressions.

Scaffolding:
- Consider challenging advanced students to justify the procedure used to divide fractions.
- $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

3. Multiply the fractions: $\frac{2}{5} \cdot \frac{3}{4}$.

$$\frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20}$$

4. **Divide the fractions:** $\frac{2}{5} \div \frac{3}{4}$.

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \cdot \frac{4}{3} = \frac{8}{15}$$

5. **Multiply the expressions:** $\frac{x+1}{x+2} \cdot \frac{3x}{x-4}$.

$$\frac{x+1}{x+2} \cdot \frac{3x}{x-4} = \frac{(x+1) \cdot 3x}{(x+2)(x-4)}$$

6. **Divide the expressions:** $\frac{x+1}{x+2} \div \frac{3x}{x-4}$.

$$\frac{x+1}{x+2} \div \frac{3x}{x-4} = \frac{x+1}{x+2} \cdot \frac{x-4}{3x} = \frac{(x+1)(x-4)}{(x+2) \cdot 3x}$$

Discussion (4 minutes)

- Is the set of rational numbers closed under multiplication? What about the set of rational expressions? Let's explore these questions together.
- Once again, we can sometimes learn more by doing less. Let's re-examine the problem in which you multiplied two fractions, but this time without doing the arithmetic.
- We have $\frac{2}{5} \cdot \frac{3}{4} = \frac{2 \cdot 3}{5 \cdot 4}$. Is the result a rational number? Why or why not?
  - *Yes. The numerator and the denominator are each the product of two integers, and the denominator is not zero.*
- Does this argument work in the general case? Take a moment to find out.
  - $\frac{\mathrm{a}}{\mathrm{b}} \cdot \frac{\mathrm{c}}{\mathrm{d}} = \frac{\mathrm{a} \cdot \mathrm{c}}{\mathrm{b} \cdot \mathrm{d}}$
  - *The numerator and denominator are each integers, and the denominator cannot be zero. This proves that the set of rational numbers is closed under multiplication.*
- Next let's examine the set of rational expressions. Is this set closed under multiplication? Let's analyze the problem from the exercise set. We showed that $\frac{x+1}{x+2} \cdot \frac{3x}{x-4} = \frac{(x+1) \cdot 3x}{(x+2)(x-4)}$. Is the result a rational expression? Why or why not?
  - *Yes. The numerator is a product of polynomials, and the denominator is the product of non-zero polynomials.*

MP.3

- Does this apply for multiplication of rational expressions in general? Explain.
  - *Yes. The product of rational expressions $x = \frac{a}{b}$ and $y = \frac{c}{d}$ is equal to $\frac{ac}{bd}$. Both ac and bd are polynomials because they are the products of polynomials, and polynomials are closed under multiplication. Also, bd is not the zero polynomial because neither b nor d are zero polynomials. Therefore, $\frac{ac}{bd}$ is a ratio of polynomials, which means that it is a rational expression.*
- Summarize this part of the discussion in one or two sentences. Share your response with a partner.
  - *Both the set of rational numbers and the set of rational expressions are closed under multiplication.*
- Okay, on to division! Try the following exercises.

Exercises 7–8 (4 minutes)

7. **Construct an argument that shows that the set of rational numbers is closed under division. That is, if x and y are rational numbers (with y nonzero) and $w = \frac{x}{y}$, prove that w must also be a rational number.**

   Let $x = \frac{a}{b}$ and let $y = \frac{c}{d}$, with both b and d nonzero.

   $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

   This is indeed a rational number. Thus, the set of rational numbers is closed under division by a nonzero number.

8. **How could you modify your argument to show that the set of rational expressions is also closed under division by a nonzero rational expression? Discuss your response with another student.**

   The only change is that a, b, c, and d represent polynomials rather than integers. The numerator and denominator of the quotient are polynomials because they both represent the product of polynomials, and polynomials are closed under multiplication. This means that the quotient is a ratio of polynomials, which fits our definition of a rational expression.

Closing (2 minutes)

- Use your notebook to briefly summarize what you learned in today's lesson.
  - *The set of rational expressions has a structure similar to the set of rational numbers. In particular, both sets are closed under addition, subtraction, multiplication, and division by a nonzero term. The properties of rational numbers are derived from properties of the integers, whereas the properties of rational expressions are derived from properties of polynomials.*
- If time permits, choose a student to share what they wrote with the class.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 10: The Structure of Rational Expressions

Exit Ticket

1. Payton says that rational expressions are not closed under addition, subtraction, multiplication, or division. His claim is shown below. Is he correct for each case? Justify your answers.

   a. $\frac{x}{2x+1} + \frac{x+1}{2x+1} = 1$, and 1 is a whole number not a rational expression.

   b. $\frac{3x-1}{2x+1} - \frac{3x-1}{2x+1} = 0$, and 0 is a whole number not a rational expression.

   c. $\frac{x-1}{x+1} \cdot \frac{x+1}{1} = x - 1$, and $x - 1$ is a whole number not a rational expression.

   d. $\frac{x-1}{x+1} \div \frac{1}{x+1} = x - 1$, and $x - 1$ is a whole number not a rational expression.

2. Simplify the following rational expressions by rewriting them with a single polynomial denominator.

   a. $\frac{3}{x-1} + \frac{2}{x}$

b. $\frac{2}{x-2} - \frac{3}{x}$

c. $\frac{x+1}{x-1} \cdot \frac{x}{x-1}$

d. $\frac{x+2}{x-1} \div \frac{x-2}{x^2-1}$

Exit Ticket Sample Solutions

1. **Payton says that rational expressions are not closed under addition, subtraction, multiplication, or division. His claim is shown below. Is he correct for each case? Justify your answers.**

   a. $\frac{x}{2x+1}+\frac{x+1}{2x+1}=1$, **and** 1 **is a whole number not a rational expression.**

   No, he is not correct. $\frac{x}{2x+1}+\frac{x+1}{2x+1}=\frac{2x+1}{2x+1}$. *The numerator and denominator are both polynomials.*

   b. $\frac{3x-1}{2x+1}-\frac{3x-1}{2x+1}=0$, **and** 0 **is a whole number not a rational expression.**

   No, he is not correct. $0=\frac{0}{1}$. *The numerator and denominator are both polynomials since integers are an example of polynomials.*

   c. $\frac{x-1}{x+1}\cdot\frac{x+1}{1}=x-1$, **and** $x-1$ **is a whole number not a rational expression.**

   No, he is not correct. $\frac{x-1}{x+1}\times\frac{x+1}{x-1}=\frac{x^2-1}{x+1}$. *The numerator and denominator are both polynomials.*

   d. $\frac{x-1}{x+1}\div\frac{1}{x+1}=x+1$, **and** $x+1$ **is a whole number not a rational expression.**

   No, he is not correct. $\frac{x-1}{x+1}\div\frac{1}{x+1}=\frac{x^2-1}{x+1}$. *The numerator and denominator are both polynomials.*

2. **Simplify the following rational expressions.**

   a. $\frac{3}{x-1}+\frac{2}{x}$

   $$\frac{5x-2}{x^2-x}$$

   b. $\frac{2}{x-2}-\frac{3}{x}$

   $$\frac{-x+6}{x^2-2x}$$

   c. $\frac{x+1}{x-1}\cdot\frac{x}{x-1}$

   $$\frac{x^2+x}{(x-1)^2}$$

   d. $\frac{x+2}{x-1}\div\frac{x-2}{x^2-1}$

   $$\frac{x^2+3x+2}{x-2}$$

Problem Set Sample Solutions

Use this space to describe any specific details about the problem set for teacher reference. Use this space to describe any specific details about the problem set for teacher reference.

1. Given $\frac{x+1}{x-2}$ and $\frac{x-1}{x^2-4}$ show that performing the following operations results in another rational expression.

a. Addition.

$$\frac{x+1}{x-2}+\frac{x-1}{x^2-4}=\frac{x^2+3x+2+x-1}{x^2-4}=\frac{x^2+4x+1}{x^2-4}$$

b. Subtraction.

$$\frac{x+1}{x-2}-\frac{x-1}{x^2-4}=\frac{x^2+3x+2-x+1}{x^2-4}=\frac{x^2+2x+3}{x^2-4}$$

c. Multiplication.

$$\frac{x+1}{x-2}\cdot\frac{x-1}{x^2-4}=\frac{x^2-1}{(x-2)(x^2-4)}$$

d. Division.

$$\frac{x+1}{x-2}\div\frac{x-1}{x^2-4}=\frac{x^2+3x+2}{x-1}$$

2. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{x-1}{x^2}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.

a. Addition.

$$\frac{x}{x^2}+\frac{-1}{x^2}=\frac{x-1}{x^2}$$

b. Subtraction.

$$\frac{x}{x^2}-\frac{1}{x^2}=\frac{x-1}{x^2}$$

c. Multiplication.

$$\frac{1}{x}\cdot\frac{x-1}{x}=\frac{x-1}{x^2}$$

d. Division.

$$\frac{1}{x}\div\frac{x}{x-1}=\frac{x-1}{x^2}$$

3. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{2x+2}{x^2-x}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.

a. Addition.

$$\frac{2x}{x^2-x}+\frac{2}{x^2-x}=\frac{2x+2}{x^2-x}$$

b. Subtraction.

$$\frac{2x}{x^2-x}-\frac{-2}{x^2-x}=\frac{2x+2}{x^2-x}$$

c. Multiplication.

$$\frac{2}{x}\cdot\frac{x+1}{x-1}=\frac{2x+2}{x^2-x}$$

d. Division.

$$\frac{2}{x}\div\frac{x-1}{x+1}=\frac{2x+2}{x^2-x}$$

4. Consider the rational expressions A, B and their quotient, $\frac{A}{B}$, where B is not equal to zero.

a. For some rational expression C, does $\frac{AC}{BC}=\frac{A}{B}$?

Whenever $C \neq 0$, $\frac{AC}{BC}=\frac{A}{B}$.

b. Let $A=\frac{x}{y}+\frac{1}{x}$ and $B=\frac{y}{x}+\frac{1}{y}$. What is the least common denominator of every term of each expression?

$$xy$$

c. Find AC, BC where C is equal to your result in part (b). Then find $\frac{AC}{BC}$. Simplify your answer.

$$AC=x^2+y$$

$$BC=y^2+x$$

$$\frac{AC}{BC}=\frac{x^2+y}{y^2+x}$$

d. Express each rational expression A, B as a single rational term; that is, as a division between two polynomials.

$$A=\frac{x^2+y}{xy}$$

$$B=\frac{y^2+x}{xy}$$

e. Write $\frac{A}{B}$ as a multiplication problem.

$$\frac{A}{B} = A \cdot \frac{1}{B}$$

f. Use your answers to parts (d) and (e) to simplify $\frac{A}{B}$.

$$\begin{aligned}\frac{A}{B} &= \frac{x^2+y}{xy} \cdot \frac{xy}{y^2+x} \\ &= \frac{x^2+y}{y^2+x}\end{aligned}$$

g. Summarize your findings. Which method do you prefer using to simplify rational expressions?

We can simplify complex rational expressions by either multiplying both the numerators and denominators by the least common denominator, or we can use the fact that division by a number is multiplication by its reciprocal. Answers may vary on preference.

5. Simplify the following rational expressions.

a. $\dfrac{\frac{1}{y}-\frac{1}{x}}{\frac{x}{y}-\frac{y}{x}}$.

$$\frac{\frac{1}{y}-\frac{1}{x}}{\frac{x}{y}-\frac{y}{x}} = \frac{\frac{x-y}{xy}}{\frac{x^2-y^2}{xy}} = \frac{1}{x+y}$$

b. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$.

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}} = \frac{1}{\frac{1}{x}-\frac{1}{y}} = \frac{xy}{y-x}$$

c. $\dfrac{\frac{1}{x^4}-\frac{1}{y^2}}{\frac{1}{x^4}+\frac{2}{x^2y}+\frac{1}{y^2}}$.

$$\frac{\frac{1}{x^4}-\frac{1}{y^2}}{\frac{1}{x^4}+\frac{2}{x^2y}+\frac{1}{y^2}} = \frac{\left(\frac{1}{x^2}+\frac{1}{y}\right)\left(\frac{1}{x^2}-\frac{1}{y}\right)}{\left(\frac{1}{x^2}+\frac{1}{y}\right)^2} = \frac{\frac{1}{x^2}-\frac{1}{y}}{\frac{1}{x^2}+\frac{1}{y}} = \frac{\frac{y-x^2}{x^2y}}{\frac{y+x^2}{x^2y}} = \frac{y-x^2}{y+x^2}$$

d. $\dfrac{\frac{1}{x-1}-\frac{1}{x}}{\frac{1}{x-1}+\frac{1}{x}}$.

$$\frac{\frac{x-x+1}{(x-1)x}}{\frac{x+x-1}{(x-1)x}}=\frac{1}{2x-1}$$

6. Find A and B that make the equation true. Verify your results.

a. $\frac{A}{x+1}+\frac{B}{x-1}=\frac{2}{x^2-1}$.

$$\frac{A(x-1)+B(x+1)}{(x+1)(x-1)}=\frac{2}{(x+1)(x-1)}.$$

Therefore,

$$A(x-1)+B(x+1)=2,$$

Let $x=1$, $A=-1$

Let $x=-1$, $B=1$

$$-\frac{1}{x+1}+\frac{1}{x-1}=\frac{2}{x^2-1}$$

b. $\frac{A}{x+3}+\frac{B}{x+2}=\frac{2x-1}{x^2+5x+6}$.

$$\frac{A(x+2)+B(x+3)}{(x+3)(x+2)}=\frac{2x-1}{(x+3)(x+2)}.$$

Therefore,

$$A(x+2)+B(x+3)=2x-1$$

Let $x=-3$, $A=7$

Let $x=-2$, $B=-5$

$$\frac{7}{x+3}-\frac{5}{x+2}=\frac{2x-1}{x^2+5x+6}$$

7. Find A, B, and C that make the equation true. Verify your result.

$\frac{Ax+B}{x^2+1}+\frac{C}{x+2}=\frac{x-1}{(x^2+1)(x+2)}$.

$$\frac{Ax+B}{x^2+1}+\frac{C}{x+2}=\frac{x-1}{(x^2+1)(x+2)},\quad (Ax+B)(x+2)+C(x^2+1)=x-1,$$

$$Ax^2+2Ax+Bx+2B+Cx^2+C=x-1$$

Therefore,

$$A+C=0,\quad 2A+B=1$$

and $2B+C=-1$

$$A=\frac{3}{5},\quad B=-\frac{1}{5},\quad C=-\frac{3}{5}$$

$$\frac{\frac{3}{5}x-\frac{1}{5}}{x^2+1}+\frac{-\frac{3}{5}}{x+2}=\frac{x-1}{(x^2+1)(x+2)}$$