



Topic B:

Rational Functions and Composition of Functions

A-APR.D.7, F-IF.C.7d, F-IF.C.9, F-BF.A.1c

Focus Standards:	A-APR.D.7	(+) Understand that rational expressions form a system analogous to the rational numbers, close under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
	F-IF.C.7	Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, showing end behavior.
	F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>
	F-BF.A.1	Write a function that describes a relationship between two quantities. c. (+) Compose functions. <i>For example, $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</i>

Instructional Days: 8

- Lesson 10:** The Structure of Rational Expressions (P)¹
- Lesson 11:** Rational Functions (P)
- Lesson 12:** End Behavior of Rational Functions (P)
- Lesson 13:** Horizontal and Vertical Asymptotes of Graphs of Rational Functions (P)
- Lesson 14:** Graphing Rational Functions (P)
- Lesson 15:** Transforming Rational Functions (E)
- Lesson 16:** Function Composition (P)
- Lesson 17:** Solving Problems by Function Composition (P)

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Previous courses have introduced students to working with rational expressions. Lesson 10 focuses on developing the idea that rational expressions form a system analogous to the rational numbers. Students use the properties of rational numbers to explore rational expressions and see that operations on rational expressions are defined in the same way as operations on rational numbers. In particular, students use the properties of integers to establish closure for the set of rational numbers. Students then use properties of polynomials to establish closure for the set of rational expressions (**A-APR.D.7**). In Module 1 of Algebra II, students simplified rational expressions and performed arithmetic operations with them, which prepared them to solve rational equations. Lesson 11 revisits this process, helping students to form conjectures about the closure property for rational functions under arithmetic operations (**A-APR.D.7**), which will lead to function composition. They review simplifying rational expressions with a focus on restricted domain values and then compare the properties of rational functions represented in different ways (**F-IF.C.9**). In Lesson 12, students look at the end behavior of rational functions numerically. Lesson 13 defines horizontal and vertical asymptotes. While students saw vertical asymptotes in Algebra II when graphing the tangent function, this is the first time that they encounter the formal definition of vertical asymptotes. Students determine horizontal and vertical asymptotes of rational functions and use technology to confirm their findings. In Lesson 14, students analyze the key features of a rational function including zeros, intercepts, asymptotes, and end behavior, and then they graph rational functions without the aid of technology (**F-IF.C.7d**). Lesson 15 extends students' work on graphs of rational functions to include transformations (**F-IF.C.7d**). Lesson 16 explores functions and their compositions, including situations where the sets representing the inputs and outputs may not be numerical. Students find the composition of functions in real-world contexts and assess the reasonableness of the compositions (**F-BF.A.1c**). Topic B concludes with Lesson 17 as students focus on composing numerical functions, including those in real-world context. The students will represent real-world relationships with equations, use those equations to create composite functions, and then use the composite functions to solve problems in both mathematical and real-world contexts (**F-IF.C.9**). Through this work students see that some compositions do not make sense or are not possible, depending on the context.

In Topic B, students use technology as a tool to understand key features of graphs (MP.5). They relate the structure of rational expressions to the graphs of rational functions in studying transformations of these graphs (MP.7). Students use mathematics to model (MP.4) as they write formulas for the surface area of spheres in terms of their diameters. They then use functions and function composition to study the relationship between a deep sea diver's depth, atmospheric pressure, and time .