Lesson 8: Curves from Geometry

Classwork

Exercises

1. Let $F(0,5)$ and $G(0,-5)$ be the foci of a hyperbola. Let the points $P(x,y)$ on the hyperbola satisfy either $PF - PG = 6$ or $PG - PF = 6$. Use the distance formula to derive an equation for this hyperbola, writing your answer in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

2. Where does the hyperbola described above intersect the y-axis?

3. Find an equation for the line that acts as a boundary for the portion of the curve that lies in the first quadrant.

4. Sketch the graph of the hyperbola described above.

Problem Set

1. For each hyperbola described below: (1) Derive an equation of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$. (2) State any x- or y-intercepts. (3) Find the equations for the asymptotes of the hyperbola.
   a. Let the foci be $A(-2,0)$ and $B(2,0)$, and let P be a point for which either $PA-PB=2$ or $PB-PA=2$.
   b. Let the foci be $A(-5,0)$ and $B(5,0)$, and let P be a point for which either $PA-PB=5$ or $PB-PA=5$.
   c. Consider $A(0,-3)$ and $B(0,3)$, and let P be a point for which either $PA-PB=2.5$ or $PB-PA=2.5$.
   d. Consider $A\left(0,-\sqrt{2}\right)$ and $B\left(0,\sqrt{2}\right)$, and let P be a point for which either $PA-PB=4$ or $PB-PA=2$.

2. Graph the hyperbolas in parts (a)–(d) in Problem 1.
   a.
   b.
   c.
   d.

3. For each value of k specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation $x^2-y^2=k$.
   a. $k=4$
   b. $k=1$
   c. $k=\frac{1}{4}$
   d. $k=0$
   e. $k=-\frac{1}{4}$
   f. $k=-1$
   g. $k=-4$
   h. Describe the hyperbolas $x^2-y^2=k$ for different values of k. Consider both positive and negative values of k, and consider values of k close to zero and far from zero.
   i. Are there any values of k so that the equation $x^2-y^2=k$ has no solution?

4. For each value of k specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation $\frac{x^2}{k}-y^2=1$.
   a. $k=-1$
   b. $k=1$
   c. $k=2$
   d. $k=4$
   e. $k=10$
   f. $k=25$

g. Describe what happens to the graph of $\frac{x^2}{k} - y^2 = 1$ as $k \to \infty$.

5. For each value of k specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation $x^2 - \frac{y^2}{k} = 1$.
   a. $k = -1$
   b. $k = 1$
   c. $k = 2$
   d. $k = 4$
   e. $k = 10$
   f. Describe what happens to the graph $x^2 - \frac{y^2}{k} = 1$ as $k \to \infty$.

6. An equation of the form $ax^2 + bx + cy^2 + dy + e = 0$ where a and c have opposite signs might represent a hyperbola.
   a. Apply the process of completing the square in both x and y to convert the equation $9x^2 - 36x - 4y^2 - 8y - 4 = 0$ to one of the standard forms for a hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$.
   b. Find the center of this hyperbola.
   c. Find the asymptotes of this hyperbola.
   d. Graph the hyperbola.

7. For each equation below, identify the graph as either an ellipse, a hyperbola, two lines, or a single point. If possible, write the equation in the standard form for either an ellipse or a hyperbola.
   a. $4x^2 - 8x + 25y^2 - 100y + 4 = 0$
   b. $4x^2 - 16x - 9y^2 - 54y - 65 = 0$
   c. $4x^2 + 8x + y^2 + 2y + 5 = 0$
   d. $-49x^2 + 98x + 4y^2 - 245 = 0$
   e. What can you tell about a graph of an equation of the form $ax^2 + bx + cy^2 + dy + e = 0$ by looking at the coefficients?