Lesson 8: Curves from Geometry

Classwork

Exercises

1. Let and be the foci of a hyperbola. Let the points on the hyperbola satisfy either or . Use the distance formula to derive an equation for this hyperbola, writing your answer in the form .
2. Where does the hyperbola described above intersect the -axis?
3. Find an equation for the line that acts as a boundary for the portion of the curve that lies in the first quadrant.
4. Sketch the graph of the hyperbola described above.

Problem Set

1. For each hyperbola described below: (1) Derive an equation of the form or .   
   (2) State any - or -intercepts. (3) Find the equations for the asymptotes of the hyperbola.
   1. Let the foci be and , and let be a point for which either or .
   2. Let the foci be and , and let be a point for which either or .
   3. Consider and , and let be a point for which either or .
   4. Consider and , and let be a point for which either or   
      .
2. Graph the hyperbolas in parts (a)–(d) in Problem 1.
3. For each value of specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation   
   .
   1. Describe the hyperbolas for different values of . Consider both positive and negative values of , and consider values of close to zero and far from zero.
   2. Are there any values of so that the equation has no solution?
4. For each value of specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation   
   .
   1. Describe what happens to the graph of as .
5. For each value of specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation   
   .
   1. Describe what happens to the graph as .
6. An equation of the form where and have opposite signs might represent a hyperbola.
   1. Apply the process of completing the square in both and to convert the equation   
       to one of the standard forms for a hyperbola: or   
      .
   2. Find the center of this hyperbola.
   3. Find the asymptotes of this hyperbola.
   4. Graph the hyperbola.
7. For each equation below, identify the graph as either an ellipse, a hyperbola, two lines, or a single point. If possible, write the equation in the standard form for either an ellipse or a hyperbola.
   1. What can you tell about a graph of an equation of the form by looking at the coefficients?