Lesson 8: Curves from Geometry

Classwork

Exercises

1. Let $F(0,5)$ and $G(0,-5)$ be the foci of a hyperbola. Let the points $P(x,y)$ on the hyperbola satisfy either $PF-PG=6$ or $PG-PF=6$. Use the distance formula to derive an equation for this hyperbola, writing your answer in the form $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$.
2. Where does the hyperbola described above intersect the $y$-axis?
3. Find an equation for the line that acts as a boundary for the portion of the curve that lies in the first quadrant.
4. Sketch the graph of the hyperbola described above.

Problem Set

1. For each hyperbola described below: (1) Derive an equation of the form $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ or $\frac{y^{2}}{b^{2}}-\frac{x^{2}}{a^{2}}=1$.
(2) State any $x$- or $y$-intercepts. (3) Find the equations for the asymptotes of the hyperbola.
	1. Let the foci be $A(-2,0)$ and $B(2,0)$, and let $P$ be a point for which either $PA-PB=2$ or $PB-PA=2$.
	2. Let the foci be $A(-5,0)$ and $B\left(5,0\right)$, and let $P$ be a point for which either $PA-PB=5$ or $PB-PA=5$.
	3. Consider $A(0,-3)$ and $B(0,3)$, and let $P$ be a point for which either $PA-PB=2.5$ or $PB-PA=2.5$.
	4. Consider $A\left(0,-\sqrt{2}\right)$ and $B\left(0,\sqrt{2}\right)$, and let $P$ be a point for which either $PA-PB=4$ or
	$PB-PA=2$.
2. Graph the hyperbolas in parts (a)–(d) in Problem 1.
3. For each value of $k$ specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation
$x^{2}-y^{2}=k$.
	1. $k=4$
	2. $k=1$
	3. $k=\frac{1}{4}$
	4. $k=0$
	5. $k=-\frac{1}{4}$
	6. $k=-1$
	7. $k=-4$
	8. Describe the hyperbolas $x^{2}-y^{2}=k$ for different values of $k$. Consider both positive and negative values of $k$, and consider values of $k$ close to zero and far from zero.
	9. Are there any values of $k$ so that the equation $x^{2}-y^{2}=k$ has no solution?
4. For each value of $k$ specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation
$\frac{x^{2}}{k}-y^{2}=1$.
	1. $k=-1$
	2. $k=1$
	3. $k=2$
	4. $k=4$
	5. $k=10$
	6. $k=25$
	7. Describe what happens to the graph of $\frac{x^{2}}{k}-y^{2}=1$ as $k\rightarrow \infty $.
5. For each value of $k$ specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation
$x^{2}-\frac{y^{2}}{k}=1$.
	1. $k=-1$
	2. $k=1$
	3. $k=2$
	4. $k=4$
	5. $k=10$
	6. Describe what happens to the graph $x^{2}-\frac{y^{2}}{k}=1$ as $k\rightarrow \infty $.
6. An equation of the form $ax^{2}+bx+cy^{2}+dy+e=0$ where $a$ and $c$ have opposite signs might represent a hyperbola.
	1. Apply the process of completing the square in both $x$ and $y$ to convert the equation
	$9x^{2}-36x-4y^{2}-8y-4=0$ to one of the standard forms for a hyperbola: $\frac{\left(x-h\right)^{2}}{a^{2}}-\frac{\left(y-k\right)^{2}}{b^{2}}=1$ or
	$\frac{\left(y-k\right)^{2}}{b^{2}}-\frac{\left(x-h\right)^{2}}{a^{2}}=1$.
	2. Find the center of this hyperbola.
	3. Find the asymptotes of this hyperbola.
	4. Graph the hyperbola.
7. For each equation below, identify the graph as either an ellipse, a hyperbola, two lines, or a single point. If possible, write the equation in the standard form for either an ellipse or a hyperbola.
	1. $4x^{2}-8x+25y^{2}-100y+4=0$
	2. $4x^{2}-16x-9y^{2}-54y-65=0$
	3. $4x^{2}+8x+y^{2}+2y+5=0$
	4. $-49x^{2}+98x+4y^{2}-245=0$
	5. What can you tell about a graph of an equation of the form $ax^{2}+bx+cy^{2}+dy+e=0$ by looking at the coefficients?