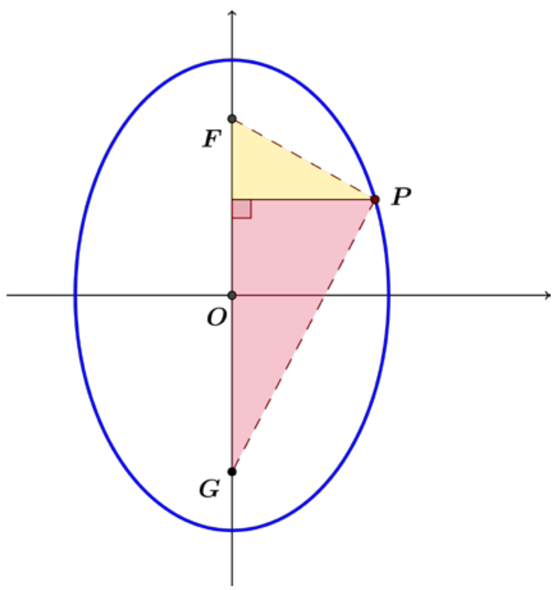
Lesson 7: Curves from Geometry

Classwork

Exercise

Points F and G are located at $(0,3)$ and $(0,-3)$. Let $P(x,y)$ be a point such that $PF + PG = 8$. Use this information to show that the equation of the ellipse is $\frac{x^2}{7} + \frac{y^2}{16} = 1$.



Problem Set

1. Derive the equation of the ellipse with the given foci F and G that passes through point P. Write your answer in standard form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
   a. The foci are $F(-2,0)$ and $G(2,0)$, and point $P(x,y)$ satisfies the condition $PF + PG = 5$.
   b. The foci are $F(-1,0)$ and $G(1,0)$, and point $P(x,y)$ satisfies the condition $PF + PG = 5$.
   c. The foci are $F(0,-1)$ and $G(0,1)$, and point $P(x,y)$ satisfies the condition $PF + PG = 4$.
   d. The foci are $F\left(-\frac{2}{3},0\right)$ and $G\left(\frac{2}{3},0\right)$, and point $P(x,y)$ satisfies the condition $PF + PG = 3$.
   e. The foci are $F(0,-5)$ and $G(0,5)$, and point $P(x,y)$ satisfies the condition $PF + PG = 12$.
   f. The foci are $F(-6,0)$ and $G(6,0)$, and point $P(x,y)$ satisfies the condition $PF + PG = 20$.

2. Recall from Lesson 6 that the semi-major axes of an ellipse are the segments from the center to the farthest vertices, and the semi-minor axes are the segments from the center to the closest vertices. For each of the ellipses in Problem 1, find the lengths a and b of the semi-major axes.

3. Summarize what you know about equations of ellipses centered at the origin with vertices $(a,0)$, $(-a,0)$, $(0,b)$, and $(0,-b)$.

4. Use your answer to Problem 3 to find the equation of the ellipse for each of the situations below.
   a. An ellipse centered at the origin with x-intercepts $(-2,0)$, $(2,0)$ and y-intercepts $(8,0)$, $(-8,0)$.
   b. An ellipse centered at the origin with x-intercepts $(-\sqrt{5},0)$, $(\sqrt{5},0)$ and y-intercepts $(3,0)$, $(-3,0)$.

5. Examine the ellipses and the equations of the ellipses you have worked with, and describe the ellipses with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the three cases $a > b$, $a = b$, and $b > a$.

6. Is it possible for $\frac{x^2}{4} + \frac{y^2}{9} = 1$ to have foci at $(-c,0)$ and $(c,0)$ for some real number c?

7. For each value of k specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation $\frac{x^2}{4} + y^2 = k$.
   a. $k = 1$
   b. $k = \frac{1}{4}$
   c. $k = \frac{1}{9}$
   d. $k = \frac{1}{16}$
   e. $k = \frac{1}{25}$
   f. $k = \frac{1}{100}$

g. Make a conjecture: Which points in the plane will satisfy the equation $\frac{x^2}{4} + y^2 = 0$?

h. Explain why your conjecture in part (g) makes sense algebraically.

i. Which points in the plane will satisfy the equation $\frac{x^2}{4} + y^2 = -1$?

8. For each value of k specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation $\frac{x^2}{k} + y^2 = 1$.

   a. $k = 1$

   b. $k = 2$

   c. $k = 4$

   d. $k = 10$

   e. $k = 25$

   f. Describe what happens to the graph of $\frac{x^2}{k} + y^2 = 1$ as $k \to \infty$.

9. For each value of k specified in parts (a)–(e), plot the set of points in the plane that satisfy the equation $x^2 + \frac{y^2}{k} = 1$.

   a. $k = 1$

   b. $k = 2$

   c. $k = 4$

   d. $k = 10$

   e. $k = 25$

   f. Describe what happens to the graph of $x^2 + \frac{y^2}{k} = 1$ as $k \to \infty$.