Lesson 4: The Binomial Theorem

Classwork

Exercises

1. Show that is a solution to the fourth degree polynomial equation .
2. Show that is a solution to the fourth degree polynomial equation .
3. Based on the patterns seen in Pascal’s triangle, what would be the coefficients of Rows 7 and 8 in the triangle? Write the coefficients of the triangle beneath the part of the triangle shown.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Row 0: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 1: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 2: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 3: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 4: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 5: |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 6: |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 7: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 8: |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. Calculate the following factorials.
2. Calculate the value of the following factorial expressions.
3. Calculate the following quantities.
   1. and
   2. , , and
   3. , , , and
   4. , , , , and
4. What patterns do you see in Exercise 5?
5. Expand the expression .
6. Expand the expression .
   1. Multiply the expression you wrote in Exercise 4 by .
   2. Multiply the expression you wrote in Exercise 4 by .
   3. How can you use the results from parts (a) and (b) to find the expanded form of the expression
7. What do you notice about your expansions for and ? Does your observation hold for other powers of
8. Use the binomial theorem to expand the following binomial expressions.

Lesson Summary

Pascal’s triangle is an arrangement of numbers generated recursively:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Row 0: |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 1: |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 2: |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 3: |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 4: |  |  |  |  |  |  |  |  |  |  |  |  |
| Row 5: |  | 1 |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

For an integer , the number is the product of all positive integers less than or equal to .   
We define .

The binomial coefficients are given by for integers and .

**The Binomial Theorem:** For any expressions and ,

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That is, the coefficients of the expanded binomial are exactly the numbers in Row of Pascal’s triangle.

Problem Set

1. Evaluate the following expressions.
2. Use the binomial theorem to expand the following binomial expressions.
3. Use the binomial theorem to expand the following binomial expressions.
   1. (Hint: .)
4. Consider the expansion of . Determine the coefficients for the terms with the powers of and shown.
5. Consider the expansion of . Determine the coefficients for the terms with the powers of and shown.
6. Consider the expansion of . Determine the coefficients for the terms with the powers of and shown.
7. Explain why the coefficient of the term that contains is in the expansion of .
8. Explain why the coefficient of the term that contains is in the expansion of .
9. Explain why the rows of Pascal’s triangle are symmetric. That is, explain why .