Lesson 2: Does Every Complex Number Have a Square Root?

Classwork

Exercises 1–6

1. Use the geometric effect of complex multiplication to describe how to calculate a square root of .
2. Calculate an estimate of a square root of .
3. Every real number has two square roots. Explain why.
4. Provide a convincing argument that every complex number must also have two square roots.
5. Explain how the polynomial identity relates to the argument that every number has two square roots.
6. What is the other square root of ?

**Example 1: Find the Square Roots of**

Find the square roots of algebraically.

Let be the square root of . Then

and

* 1. Expand the left side of this equation.
  2. Equate the real and imaginary parts, and solve for and .
  3. What are the square roots of ?

Exercises 7–9

1. Use the method in Example 1 to find the square roots of .
2. Find the square roots of each complex number.
3. Show that if is a square root of , then is a square root of the conjugate of , .
   1. Explain why .
   2. What do and equal in terms of and ?
   3. Calculate . What is the real part, and what is the imaginary part?
   4. Explain why .

Lesson Summary

The square roots of a complex number will be of the form and and can be found by solving the equations and .

Problem Set

Find the two square roots of each complex number by creating and solving polynomial equations.



A *Pythagorean triple* is a set of three positive integers , , and such that . Thus, these integers can be the lengths of the sides of a right triangle.

1. Show algebraically that for positive integers and , if

then ,

1. Select two integers and , use the formulas in Problem 8 to find , , and , and then show those numbers satisfy the equation .
2. Use the formulas from Problem 8, and find values for and that give the following famous triples.
3. Is it possible to write the Pythagorean triple in the form , , for some integers and ? Verify your answer.
4. Choose your favorite Pythagorean triple that has and sharing only as a common factor, for example , , or ,… Find the square of the length of a square root of ; that is, find , where is a square root of . What do you observe?