Lesson 1: Solutions to Polynomial Equations

Classwork

Opening Exercise

How many solutions are there to the equation ? Explain how you know.

Example 1: Prove That A Quadratic Equation Has Only Two Solutions Over The Set Of Complex Numbers

Prove that and are the only solutions to the equation

Let be a complex number so that .

* 1. Substitute for in the equation .
  2. Rewrite both sides in standard form for a complex number.
  3. Equate the real parts on each side of the equation and equate the imaginary parts on each side of the equation.
  4. Solve for and and find the solutions for .

Exercises

Find the product.

Write each of the following quadratic expressions as the product of two linear factors.

1. Can a quadratic polynomial equation with real coefficients have one real solution and one complex solution? If so, give an example of such an equation. If not, explain why not.

Recall from Algebra II that every quadratic expression can be written as a product of two linear factors, that is,

where and are solutions of the polynomial equation .

1. Solve each equation by factoring, and state the solutions.
2. Give an example of a quadratic equation with as one of its solutions.
3. A quadratic polynomial equation with real coefficients has a complex solution of the form with . What must its other solution be, and why?
4. Write the left side of each equation as a product of linear factors, and state the solutions.
5. Consider the polynomial .
   1. Graph and find the real zero of polynomial
   2. Write as a product of linear factors.
   3. What are the solutions to the equation ?
6. Malaya was told that the volume of a box that is a cube is cubic inches. She knows the formula for the volume of a cube with side length is , so she models the volume of the box with the equation   
   .
   1. Solve this equation for .
   2. Malaya shows her work to Tiffany and tells her that she has found three different values for the side length of the box. Tiffany looks over Malaya’s work and sees that it is correct but explains to her that there is only one valid answer. Help Tiffany explain which answer is valid and why.
7. Consider the polynomial
   1. Graph and state the number of real zeros of .
   2. Verify that is a zero of
   3. Given that is a zero of , state another zero of
   4. Given that and are also zeros of , explain why polynomial cannot possibly have any real zeros.
   5. What is the solution set to the equation ?
8. Think of an example of a sixth degree polynomial equation that when written in standard form has integer coefficients, four real number solutions, and two imaginary number solutions. How can you be sure your equation will have integer coefficients?

Lesson Summary

Relevant Vocabulary

Polynomial Function: Given a polynomial expression in one variable, a *polynomial function in one variable* is a function such that for each real number in the domain, is the value found by substituting the number into all instances of the variable symbol in the polynomial expression and evaluating.

It can be shown that if a function is a polynomial function, then there is some nonnegative integer and collection of real numbers , , ,, with such that the function satisfies the equation

for every real number in the domain, which is called the *standard form of the polynomial function.* The function , where can be any real number, is an example of a function written in standard form.

Degree of a Polynomial Function: The *degree of a polynomial function* is the degree of the polynomial expression used to define the polynomial function. The degree is the highest degree of its terms.

The degree of is 3, but the degree of is because when is put into standard form, it is .

Zeros or Roots of a Function: A *zero* (or *root*) of a function is a number of the domain such that . A zero of a function is an element in the solution set of the equation .

Given any two polynomial functions and , the solution set of the equation can be quickly found by solving the two equations and and combining the solutions into one set.

A number is zero of a polynomial function with multiplicity if the factored form of contains .

Every polynomial function of degree , for , has zeros over the complex numbers, counted with multiplicity. Therefore, such polynomials can always be factored into linear factors.

Problem Set

1. Find all solutions to the following quadratic equations, and write each equation in factored form.
2. The following cubic equations all have at least one real solution. Find the remaining solutions.
3. Find the solutions of the following equations.