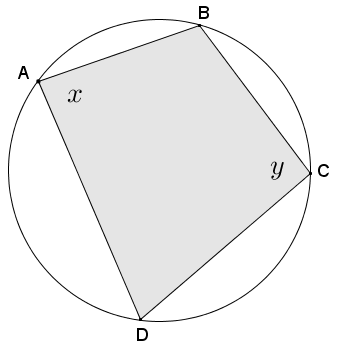
Lesson 20: Cyclic Quadrilaterals

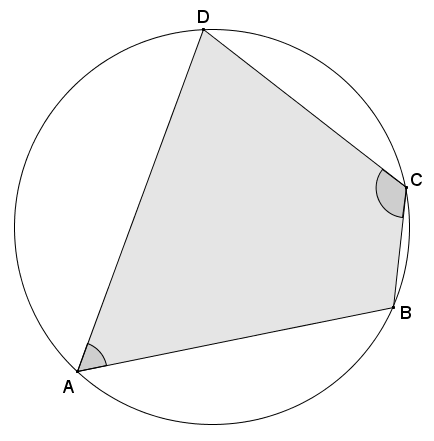
Classwork

Opening Exercise

Given cyclic quadrilateral shown in the diagram, prove that .

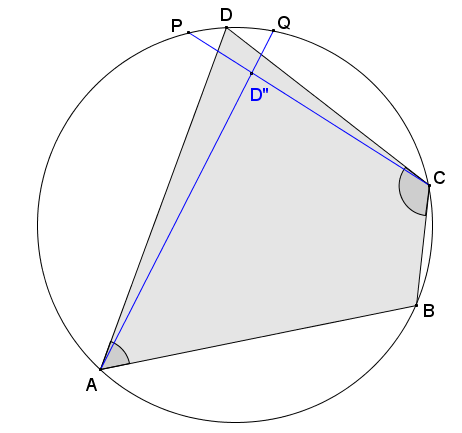
**Example 1:**

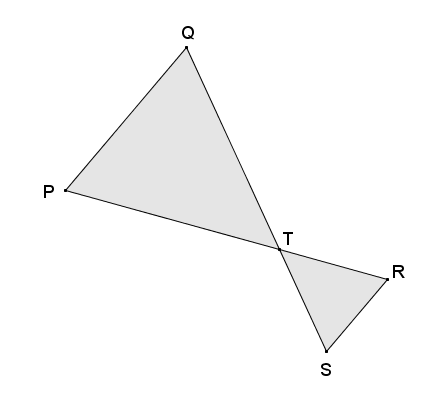
Given quadrilateral with , prove that quadrilateral is cyclic; in other words, prove that points and lie on the same circle.

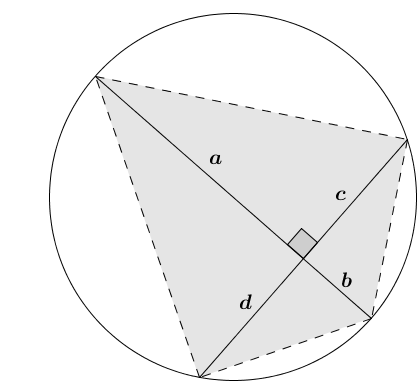


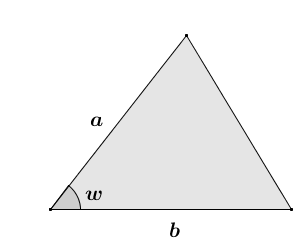
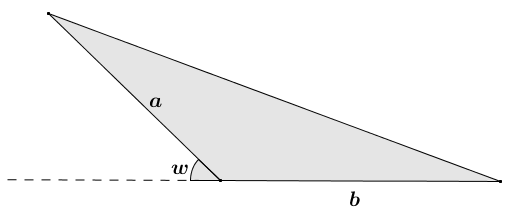
Exercises

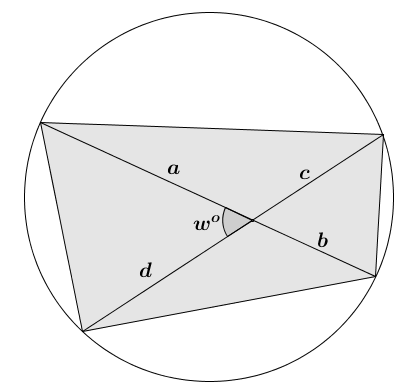
1. Assume that vertex lies inside the circle as shown in the diagram. Use a similar argument to Example 1 to show that vertex cannot lie inside the circle.

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1. Quadrilateral is a cyclic quadrilateral. Explain why .
2. A cyclic quadrilateral has perpendicular diagonals. What is the area of the quadrilateral in terms of , ,, and as shown?



1. Show that the triangle in the diagram has area .
2. Show that the triangle with obtuse angle has area .
3. Show that the area of the cyclic quadrilateral shown in the diagram is .



Lesson Summary

Theorems:

Given a convex quadrilateral, the quadrilateral is cyclic if and only if one pair of opposite angles is supplementary.

The area of a triangle with side lengths and and acute included angle with degree measure :

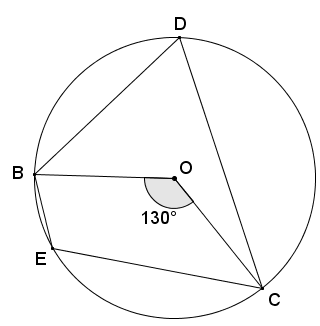
The area of a cyclic quadrilateral whose diagonals and intersect to form an acute or right angle with degree measure :

Relevant Vocabulary

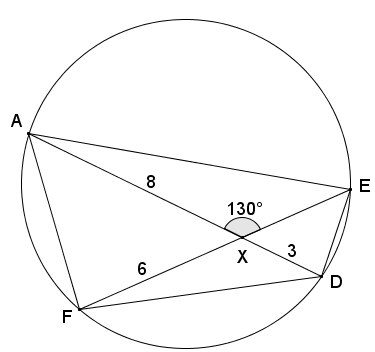
**Cyclic Quadrilateral:** A quadrilateral inscribed in a circle is called a *cyclic quadrilateral*.

Problem Set

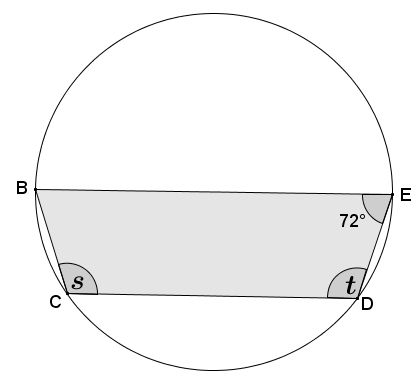
1. Quadrilateral is cyclic, is the center of the circle, and . Find .



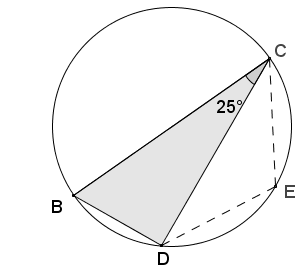
1. Quadrilateral is cyclic, , , , and . Find the area of quadrilateral .

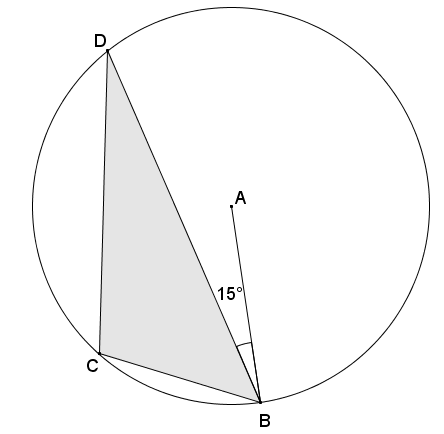
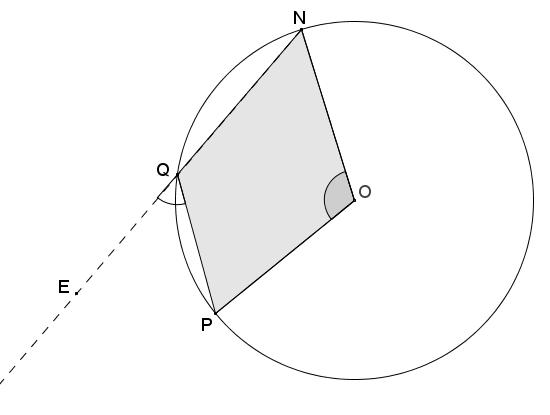


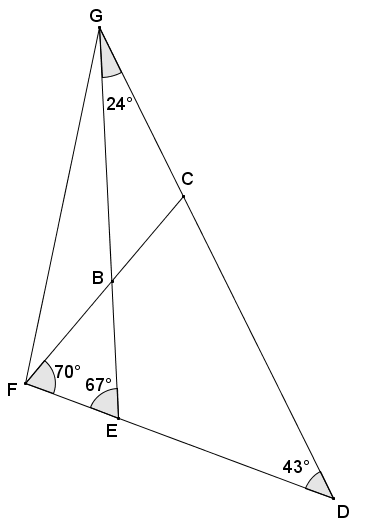
1. In the diagram below, , and . Find the value of and .

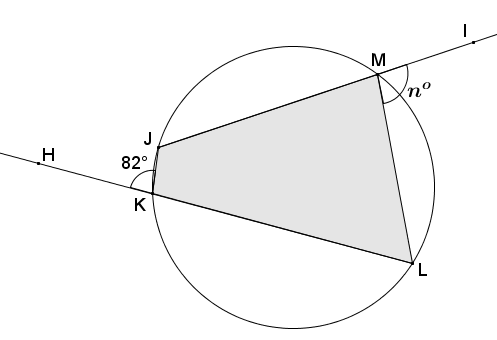
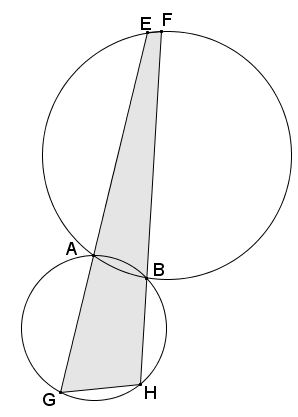
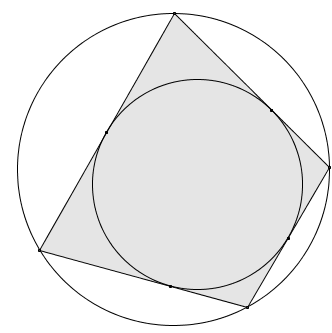
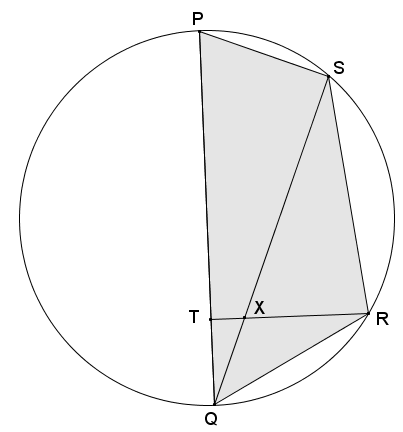


1. In the diagram below, is the diameter, , and . Find .



1.  In circle, . Find .
2. Given the diagram below, is the center of the circle. If , find .
3. Given the angle measures as indicated in the diagram below, prove that vertices ,,, and lie on a circle.



1. In the diagram below, quadrilateral is cyclic. Find the value of .
2. Do all four perpendicular bisectors of the sides of a cyclic quadrilateral pass through a common point? Explain.
3. The circles in the diagram below intersect at points and . If and , find and .
4. A quadrilateral is called *bicentric* if it is both cyclic and possesses an inscribed circle. (See diagram to the right.)
   1. What can be concluded about the opposite angles of a bicentric quadrilateral? Explain.
   2. Each side of the quadrilateral is tangent to the inscribed circle. What does this tell us about the segments contained in the sides of the quadrilateral?
   3. Based on the relationships highlighted in part (b), there are four pairs of congruent segments in the diagram. Label segments of equal length with , and .
   4. What do you notice about the opposite sides of the bicentric quadrilateral?
5. Quadrilateral is cyclic such that is the diameter of the circle. If , prove that is a right angle, and show that ,,, and lie on a circle.