Lesson 19: Equations for Tangent Lines to Circles

Classwork

Opening Exercise

A circle of radius 5 passes through points $A(-3,3)$ and $B(3,1)$.

a. What is the special name for segment AB?

b. How many circles can be drawn that meet the given criteria? Explain how you know.

c. What is the slope of $\overline{AB}$?

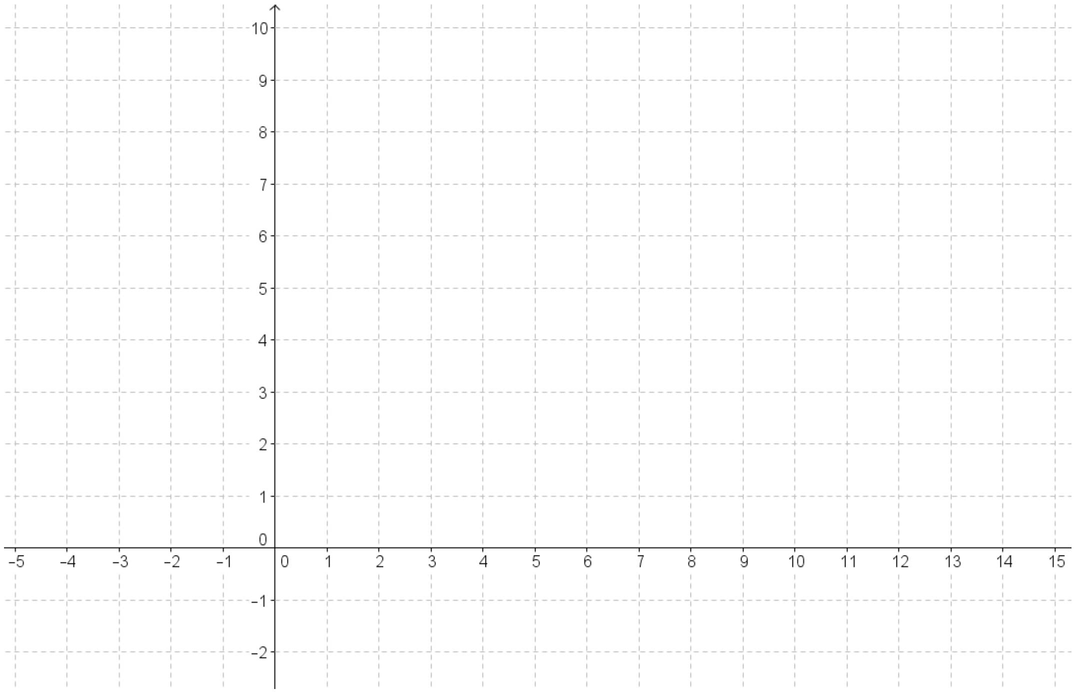
d. Find the midpoint of $\overline{AB}$.

e. Find the equations of the line containing a diameter of the given circle perpendicular to $\overline{AB}$.

f. Is there more than one answer possible for part (e)?

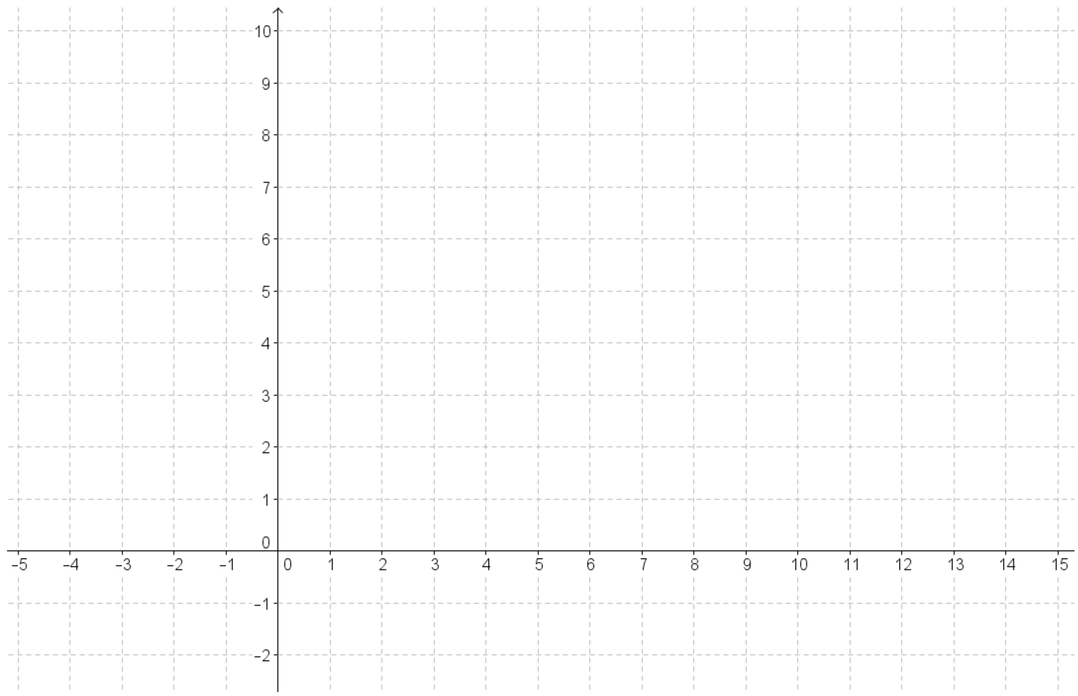
Example 1

Consider the circle with equation $(x-3)^2 + (y-5)^2 = 20$. Find the equations of two tangent lines to the circle that each have slope $-\frac{1}{2}$.



Exercise 1

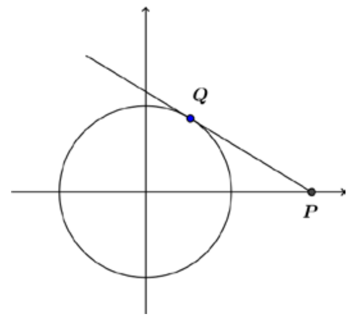
Consider the circle with equation $(x-4)^2+(y-5)^2=20$. Find the equations of two tangent lines to the circle that each have slope 2.



Example 2

Refer to the diagram below.

Let $p > 1$. What is the equation of the tangent line to the circle $x^2 + y^2 = 1$ through the point $(p, 0)$ on the x-axis with a point of tangency in the upper half-plane?



Exercises

2. Use the same diagram from Example 2 above, but label the point of tangency in the lower half-plane as Q'.

   a. What are the coordinates of Q'?

   b. What is the slope of $\overline{OQ'}$?

   c. What is the slope of $\overline{Q'P}$?

   d. Find the equation of the second tangent line to the circle through $(p, 0)$.

3. Show that a circle with equation $(x-2)^2+(y+3)^2=160$ has two tangent lines with equations $y+15=\frac{1}{3}(x-6)$ and $y-9=\frac{1}{3}(x+2)$.

4. Could a circle given by the equation $(x-5)^2+(y-1)^2=25$ have tangent lines given by the equations $y-4=\frac{4}{3}(x-1)$ and $y-5=\frac{3}{4}(x-8)$? Explain how you know.

Lesson Summary

Theorems

A tangent line to a circle is perpendicular to the radius of the circle drawn to the point of tangency.

Relevant Vocabulary

TANGENT TO A CIRCLE. A *tangent line to a circle* is a line in the same plane that intersects the circle in one and only one point. This point is called the *point of tangency*.

Problem Set

1. Consider the circle $(x-1)^2+(y-2)^2=16$. There are two lines tangent to this circle having a slope of 0.
   a. Find the coordinates of the points of tangency.
   b. Find the equations of the two tangent lines.

2. Consider the circle $x^2-4x+y^2+10y+13=0$. There are two lines tangent to this circle having a slope of $\frac{2}{3}$.
   a. Find the coordinates of the two points of tangency.
   b. Find the equations of the two tangent lines.

3. What are the coordinates of the points of tangency of the two tangent lines through the point $(1,1)$ each tangent to the circle $x^2+y^2=1$?

4. What are the coordinates of the points of tangency of the two tangent lines through the point $(-1,-1)$ each tangent to the circle $x^2+y^2=1$?

5. What is the equation of the tangent line to the circle $x^2+y^2=1$ through the point $(6,0)$?

6. D'Andre said that a circle with equation $(x-2)^2+(y-7)^2=13$ has a tangent line represented by the equation $y-5=-\frac{3}{2}(x+1)$. Is he correct? Explain.

7. Kamal gives the following proof that $y - 1 = \frac{8}{9}(x + 10)$ is the equation of a line that is tangent to a circle given by $(x + 1)^2 + (y - 9)^2 = 145$.

   The circle has center $(-1,9)$ and radius 12. The point $(-10,1)$ is on the circle because

   $$(-10 + 1)^2 + (1 - 9)^2 = (-9)^2 + (-8)^2 = 145.$$

   The slope of the radius is $\frac{9-1}{-1-10} = \frac{8}{9}$; therefore, the equation of the tangent line is $y - 1 = \frac{8}{9}(x + 10)$.

   a. Kerry said that Kamal has made an error. What was Kamal's error? Explain what he did wrong.

   b. What should the equation for the tangent line be?

8. Describe a similarity transformation that maps a circle given by $x^2 + 6x + y^2 - 2y = 71$ to a circle of radius 3 that is tangent to both axes in the first quadrant.