

Lesson 18: Recognizing Equations of Circles

Classwork

Opening Exercise

- a. Express this as a trinomial: $(x - 5)^2$.

	x	3	
x	x^2	$3x$	$=$
3	$3x$	$?$	
	x	3	
	x^2	$3x$	$=$
3	$3x$	9	

- b. Express this as a trinomial: $(x + 4)^2$.

- c. Factor the trinomial : $x^2 + 12x + 36$.

- d. Complete the square to solve the following equation: $x^2 + 6x = 40$.

Example 1

The following is the equation of a circle with radius 5 and center (1,2). Do you see why?

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$$

Exercise

1. Rewrite the following equations in the form $(x - a)^2 + (y - b)^2 = r^2$.

a. $x^2 + 4x + 4 + y^2 - 6y + 9 = 36$

b. $x^2 - 10x + 25 + y^2 + 14y + 49 = 4$

Example 2

What is the center and radius of the following circle?

$$x^2 + 4x + y^2 - 12y = 41$$

Exercises

2. Identify the center and radius for each of the following circles.

a. $x^2 - 20x + y^2 + 6y = 35$

b. $x^2 - 3x + y^2 - 5y = \frac{19}{2}$

3. Could the circle with equation $x^2 - 6x + y^2 - 7 = 0$ have a radius of 4? Why or why not?
4. Stella says the equation $x^2 - 8x + y^2 + 2y = 5$ has a center of $(4, -2)$ and a radius of $\sqrt{22}$. Is she correct? Why or why not?

Example 3

Could $x^2 + y^2 + Ax + By + C = 0$ represent a circle?

Exercise

5. Identify the graphs of the following equations as a circle, a point, or an empty set.

a. $x^2 + y^2 + 4x = 0$

b. $x^2 + y^2 + 6x - 4y + 15 = 0$

c. $2x^2 + 2y^2 - 5x + y + \frac{13}{4} = 0$

Problem Set

1. Identify the center and radii of the following circles.
 - a. $(x + 25) + y^2 = 1$
 - b. $x^2 + 2x + y^2 - 8y = 8$
 - c. $x^2 - 20x + y^2 - 10y + 25 = 0$
 - d. $x^2 + y^2 = 19$
 - e. $x^2 + x + y^2 + y = -\frac{1}{4}$
2. Sketch a graph of the following equations.
 - a. $x^2 + y^2 + 10x - 4y + 33 = 0$
 - b. $x^2 + y^2 + 14x - 16y + 104 = 0$
 - c. $x^2 + y^2 + 4x - 10y + 29 = 0$
3. Chante claims that two circles given by $(x + 2)^2 + (y - 4)^2 = 49$ and $x^2 + y^2 - 6x + 16y + 37 = 0$ are externally tangent. She is right. Show that she is.
4. Draw a circle. Randomly select a point in the interior of the circle; label the point A . Construct the greatest radius circle with center A that lies within the circular region defined by the original circle. Hint: Draw a line through the center, the circle, and point A .