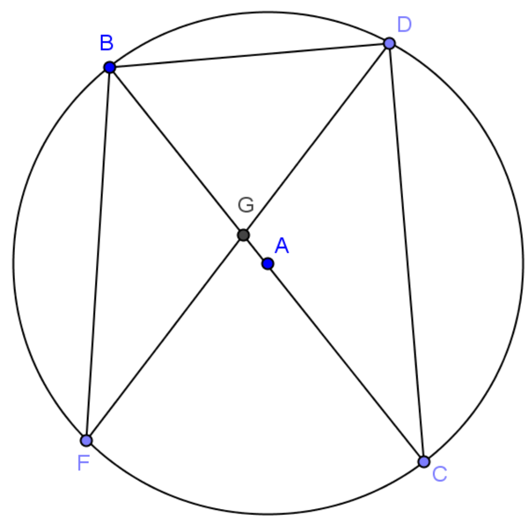
Lesson 13: The Inscribed Angle Alternate a Tangent Angle

Classwork

Opening Exercise

1. In circle A, $m\widehat{BD} = 56°$, and $\overline{BC}$ is a diameter. Find the listed measure, and explain your answer.



a. $m\angle BDC$

b. $m\angle BCD$

c. $m\angle DBC$

d. $m\angle BFG$

e. $m\widehat{BC}$

f. $m\widehat{DC}$

g. Is the $m\angle BGD = 56°$? Explain.

h. How do you think we could determine the measure of $\angle BGD$?

Example 1

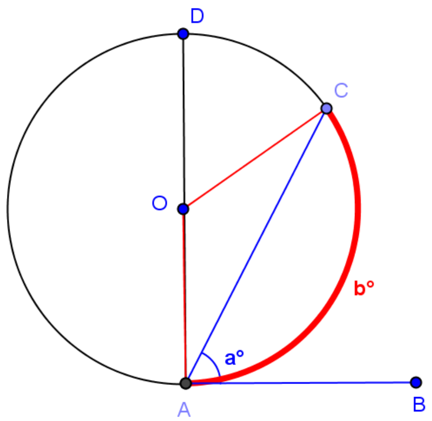


Diagram 1

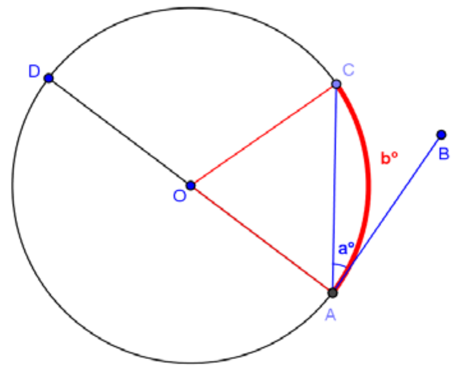


Diagram 2

Examine the diagrams shown. Develop a conjecture about the relationship between a and b.

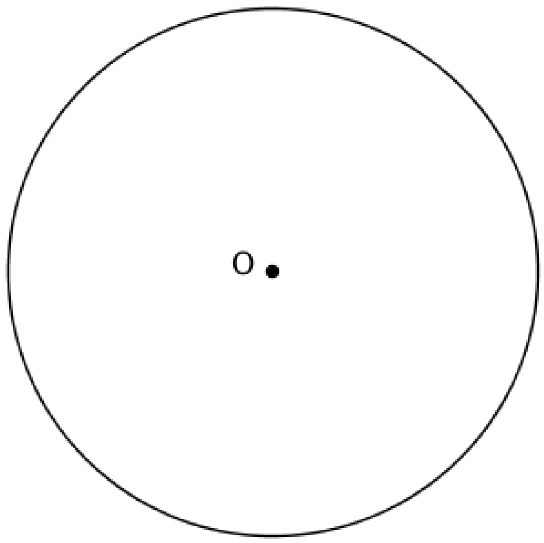
Test your conjecture by using a protractor to measure a and b.

	a	b
Diagram 1		
Diagram 2		

Do your measurements confirm the relationship you found in your homework?

If needed, revise your conjecture about the relationship between a and b:

Now test your conjecture further using the circle below.

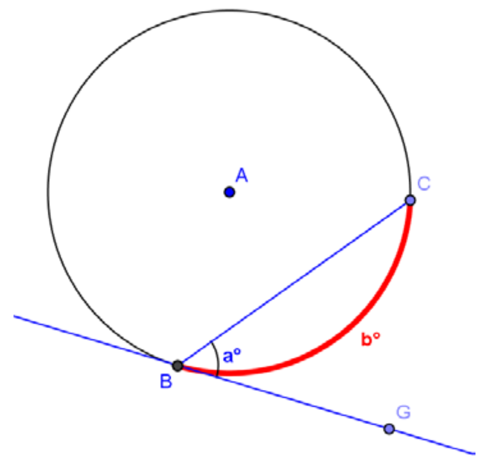


a	b

Now, we will prove your conjecture, which is stated below as a theorem.

THE TANGENT-SECANT THEOREM: Let A be a point on a circle, let $\overrightarrow{AB}$ be a tangent ray to the circle, and let C be a point on the circle such that $\overleftrightarrow{AC}$ is a secant to the circle. If $a = m\angle BAC$ and b is the angle measure of the arc intercepted by $\angle BAC$, then $a = \frac{1}{2}b$.

Given circle A with tangent $\overleftrightarrow{BG}$, prove what we have just discovered using what you know about the properties of a circle and tangent and secant lines.



a. Draw triangle ABC. What is the measure of $\angle BAC$? Explain.

b. What is the measure of $\angle ABG$? Explain.

c. Express the measure of the remaining two angles of triangle ABC in terms of "a" and explain.

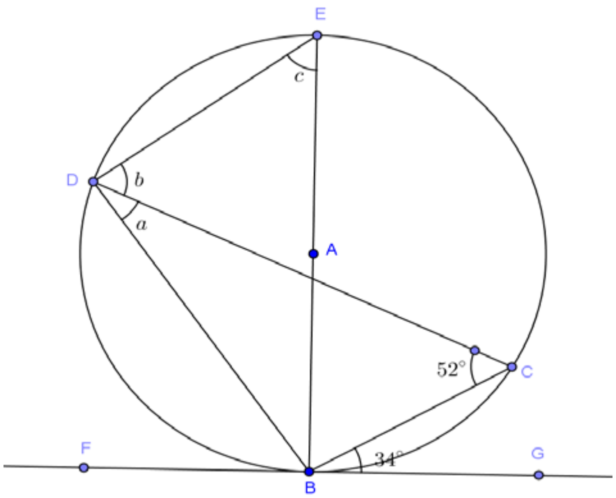
d. What is the measure of $\angle BAC$ in terms of "a"? Show how you got the answer.

e. Explain to your neighbor what we have just proven.

Exercises

Find x, y, a, b, and/or c.

1.



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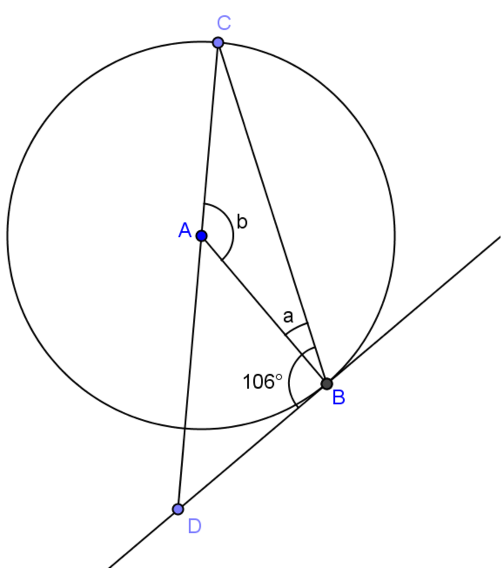

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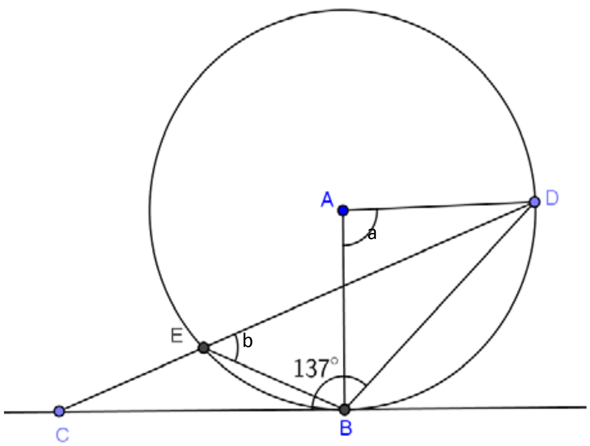

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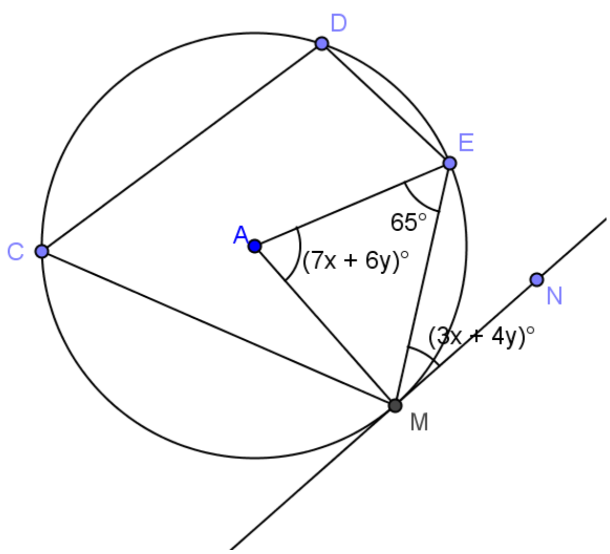

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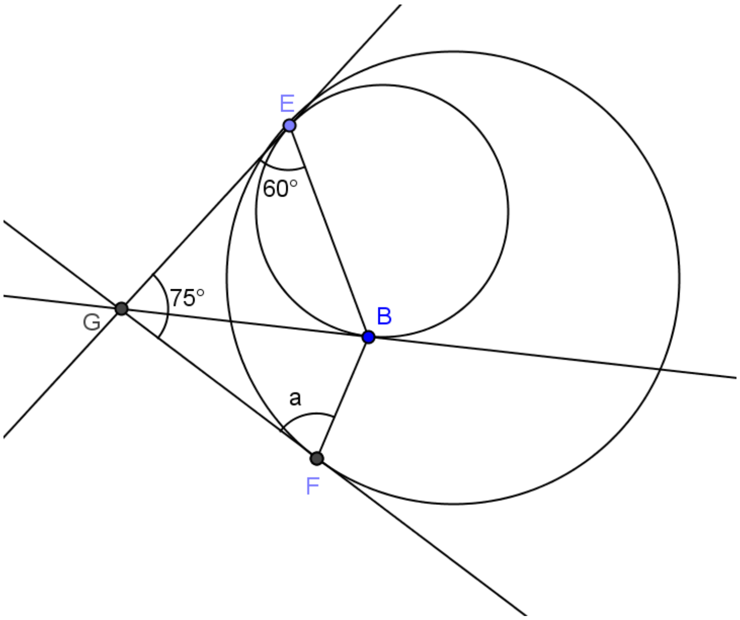


Lesson Summary

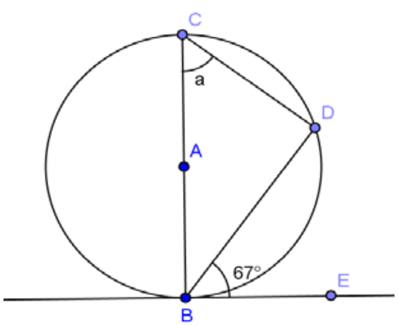
THEOREMS:

- **CONJECTURE:** Let A be a point on a circle, let $\overrightarrow{\mathrm{AB}}$ be a tangent ray to the circle, and let C be a point on the circle such that $\overleftrightarrow{\mathrm{AC}}$ is a secant to the circle. If $\mathrm{a} = \mathrm{m}\angle\mathrm{BAC}$ and b is the angle measure of the arc intercepted by $\angle\mathrm{BAC}$, then $\mathrm{a} = \frac{1}{2}\mathrm{b}$.
- **THE TANGENT-SECANT THEOREM:** Let A be a point on a circle, let $\overrightarrow{\mathrm{AB}}$ be a tangent ray to the circle, and let C be a point on the circle such that $\overleftrightarrow{\mathrm{AC}}$ is a secant to the circle. If $\mathrm{a} = \mathrm{m}\angle\mathrm{BAC}$ and b is the angle measure of the arc intercepted by $\angle\mathrm{BAC}$, then $\mathrm{a} = \frac{1}{2}\mathrm{b}$.
- Suppose $\overline{\mathrm{AB}}$ is a chord of circle C, and $\overline{\mathrm{AD}}$ is a tangent segment to the circle at point A. If E is any point other than A or B in the arc of C on the opposite side of $\overline{\mathrm{AB}}$ from D, then $\mathrm{m}\angle\mathrm{BEA} = \mathrm{m}\angle\mathrm{BAD}$.

Problem Set

In Problems 1–9, solve for a, b, and/or c.

1.



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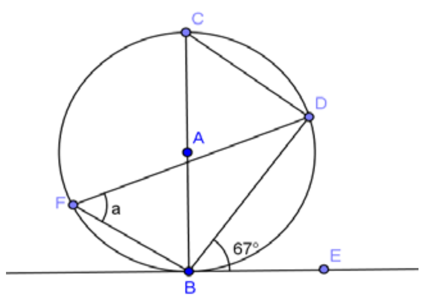

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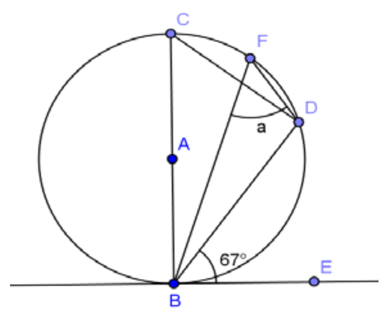

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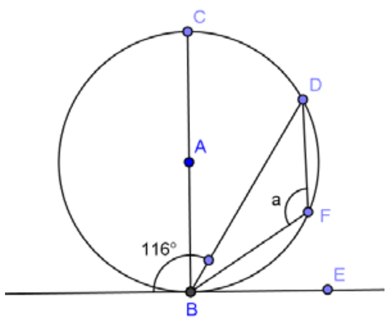

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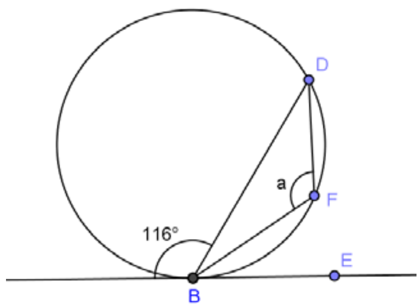

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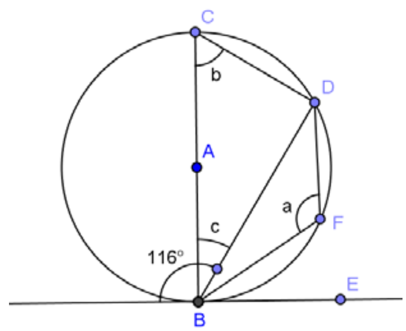

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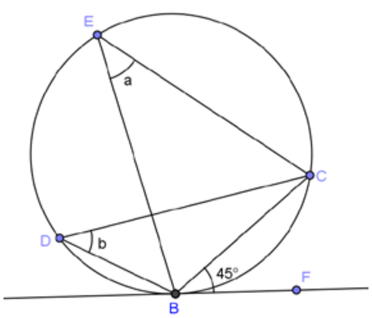

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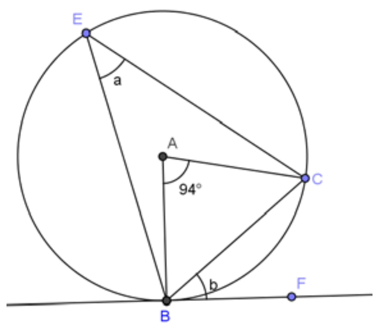

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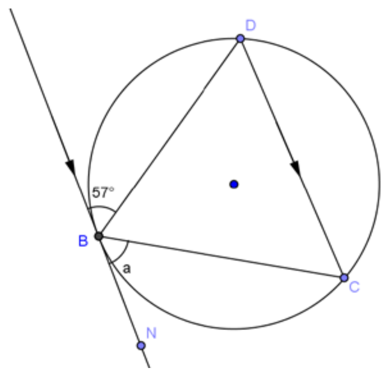


10. $\overleftrightarrow{BH}$ is tangent to circle A. $\overline{DF}$ is a diameter. Find

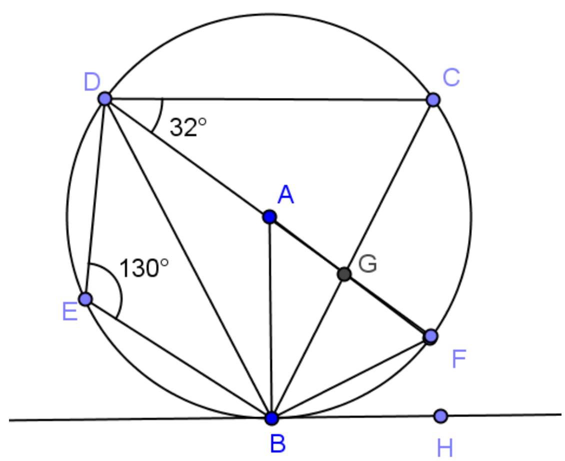
a. $m\angle BCD$

b. $m\angle BAF$

c. $m\angle BDA$

d. $m\angle FBH$

e. $m\angle BGF$



11. $\overline{BG}$ is tangent to circle A. $\overline{BE}$ is a diameter. Prove: (i) $f = a$ and (ii) $d = c$