Lesson 13: The Inscribed Angle Alternate a Tangent Angle

Classwork

Opening Exercise

1. In circle $A$, $m\hat{BD}=56°,$ and $\overbar{BC}$ is a diameter. Find the listed measure, and explain your answer.
	1. $m∠BDC$
	2. $m∠BCD$
	3. $m∠DBC$
	4. $m∠BFG$
	5. $m\hat{BC}$
	6. $m\hat{DC} $
	7. Is the $m∠BGD=56°$? Explain.
	8. How do you think we could determine the measure of $∠BGD$?

**Example 1**

Diagram 1

Diagram 2

Examine the diagrams shown. Develop a conjecture about the relationship between $a$ and $b$.

Test your conjecture by using a protractor to measure $a$ and $b$.

|  |  |  |
| --- | --- | --- |
|  | $$a$$ | $$b$$ |
| Diagram 1 |  |  |
| Diagram 2 |  |  |

Do your measurements confirm the relationship you found in your homework?

If needed, revise your conjecture about the relationship between $a$ and $b$:

Now test your conjecture further using the circle below.



|  |  |
| --- | --- |
| $$a$$ | $$b$$ |
|  |  |

Now, we will prove your conjecture, which is stated below as a theorem.

**The tangent-secant theorem**: Let$ A$ be a point on a circle, let $\vec{AB}$ be a tangent ray to the circle, and let $C$ be a point on the circle such that $\overleftrightarrow{AC}$ is a secant to the circle. If $a=m∠BAC$ and $b$ is the angle measure of the arc intercepted by $∠BAC$, then $a=\frac{1}{2}b$.

Given circle $A$ with tangent $\overleftrightarrow{BG}$, prove what we have just discovered using what you know about the properties of a circle and tangent and secant lines.

* + - * 1. Draw triangle $ABC$. What is the measure of $∠BAC$? Explain.
				2. What is the measure of $∠ABG$? Explain.
				3. Express the measure of the remaining two angles of triangle $ABC$ in terms of “$a$” and explain.
				4. What is the measure of $∠BAC$ in terms of “$a$”? Show how you got the answer.
				5. Explain to your neighbor what we have just proven.

Exercises

Find $x, y, a, b, $and/or $c$.

1.

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2.



3.

a

b

4.



5.



Problem Set

Lesson Summary

Theorems:

* **Conjecture:**Let$ A$ be a point on a circle, let $\vec{AB}$ be a tangent ray to the circle, and let $C$ be a point on the circle such that $\overleftrightarrow{AC}$ is a secant to the circle. If $a=m∠BAC$ and $b$ is the angle measure of the arc intercepted by $∠BAC$, then $a=\frac{1}{2}b$.
* The tangent-secant theorem:Let$ A$ be a point on a circle, let $\vec{AB}$ be a tangent ray to the circle, and let $C$ be a point on the circle such that $\overleftrightarrow{AC}$ is a secant to the circle. If $a=m∠BAC$ and $b$ is the angle measure of the arc intercepted by $∠BAC$, then $a=\frac{1}{2}b$.
* Suppose $\overbar{AB}$ is a chord of circle $C$, and $\overbar{AD}$ is a tangent segment to the circle at point $A$. If $E$ is any point other than $A$ or $B$ in the arc of $C$ on the opposite side of $\overbar{AB}$ from $D$, then $m∠BEA=m∠BAD$.

In Problems 1–9, solve for $a, b$, and/or $c.$

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1. $\overleftrightarrow{BH} $is tangent to circle $A$. $\overbar{DF}$ is a diameter. Find
	1. $m∠BCD$
	2. $m∠BAF$
	3. $m∠BDA$
	4. $m∠FBH$
	5. $m∠BGF$
2. $\overbar{BG}$ is tangent to circle $A$. $\overbar{BE}$ is a diameter. Prove: (i) $f=a$ and (ii)$ d=c$

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