Lesson 10: Unknown Length and Area Problems

Student Outcomes

- Students apply their understanding of *arc length* and *area of sectors* to solve problems of unknown area and length.

Lesson Notes

This lesson continues the work started in Lesson 9 as students solve problems on arc length and area of sectors. The lesson is intended to be 45 minutes of problem solving with a partner. Problems vary in level of difficulty and can be assigned specifically based on student understanding. The problem set can be used in class for some students or assigned as homework. Students who need to focus on a small number of problems could finish the other problems at home. Teachers may choose to model two or three problems with the entire class.

Exercise 4 is a modeling problem highlighting **G-MG.A.1** and MP.4.

Classwork

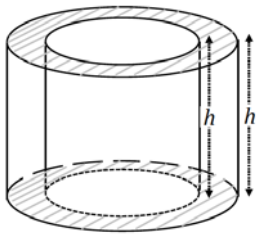
Begin with a quick whole class discussion of an annulus. Project the figure on the right on the board.

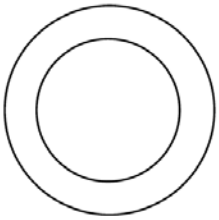
Opening Exercise (3 minutes)

Opening Exercise

In the following figure, a cylinder is carved out from within another cylinder of the same height; the bases of both cylinders share the same center.

a. Sketch a cross section of the figure parallel to the base.

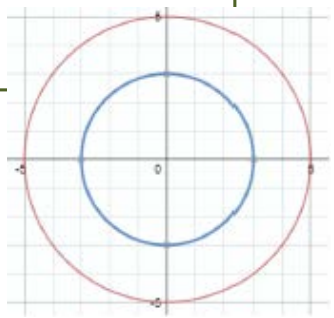




Confirm that students' sketch is correct before allowing them to proceed to part (b).

Scaffolding:

- Post area of sector and arc length formulas for easy reference.
- A review of compound figures may be required before this lesson.
- Scaffold the task by asking students to compute the area of the circle with radius r, then the circle with radius s, and then ask how the shaded region is related to the two circles.
- Use an example with numerical values for s and r on the coordinate plane, and ask students to estimate the area first (see example below).



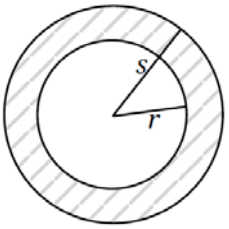
MP.2 & MP.7

b. Mark and label the shorter of the two radii as r and the longer of the two radii s.
Show how to calculate the area of the shaded region and explain the parts of the expression.

$Area(shaded) = \pi(s^2 - r^2)$

s = radius of outer circle

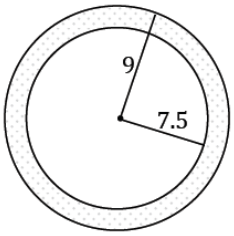
r = radius of inner circle



The figure you sketched in part (b) is called an *annulus*; it is a ring shaped region or the region lying between two concentric circles. In Latin, annulus means "little ring."

Exercises (35 minutes)

1. Find the area of the following annulus.



$Area(shaded) = \pi(9^2 - 7.5^2)$

$Area(shaded) = 24.75\pi$

The area of the annulus is 24.75π units^2.

2. The larger circle of an annulus has a diameter of 10 cm, and the smaller circle has a diameter of 7.6 cm. What is the area of the annulus?

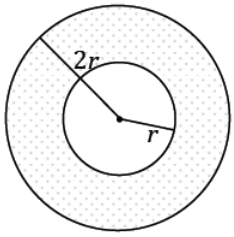
The radius of the larger circle is 5 cm, and the radius of the smaller circle is 3.8 cm.

$Area(shaded) = \pi(5^2 - 3.8^2)$

$Area(shaded) = 10.56\pi$

The area of the annulus is 10.56π cm^2.

3. In the following annulus, the radius of the larger circle is twice the radius of the smaller circle. If the area of the following annulus is 12π units^2, what is the radius of the larger circle?



$Area(shaded)$:

$$\pi((2r)^2 - r^2) = 12\pi$$
$$3r^2 = 12$$
$$r = 2$$

The radius of the larger circle is twice the radius of the smaller circle or $2(2) = 4$; the radius of the larger circle is 4 units.

MP.4

4. An ice cream shop wants to design a super straw to serve with their extra thick milkshakes that is double the width and thickness of a standard straw. A standard straw is 8 mm in diameter and 0.5 mm thick.

   a. What is the cross-sectional (parallel to the base) area of the new straw (round to the nearest hundredth)?

   $47.12\ mm^2$

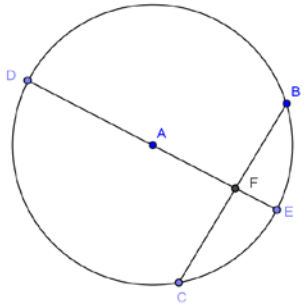
   b. If the new straw is 23 mm long, what is the maximum volume of milkshake that can be in the straw at one time (round to the nearest hundredth)?

   $3,540.57\ mm^3$

   c. A large milkshake is 32 ounces (approximately 950 mL). If Corbin withdraws the full capacity of a straw 10 times a minute, what is the minimum amount of time that it will take him to drink the milkshake (round to the nearest minute)?

   27 *minutes*

5. In the circle given, $\overline{ED}$ is the diameter and is perpendicular to chord $\overline{CB}$. $DF = 8$ cm and $FE = 2$ cm. Find AC, BC, $m\angle CAB$, the arc length of $\overset{\frown}{CEB}$, and the area of sector $\overset{\frown}{CEB}$ (round to the nearest hundredth, if necessary).



   $AC = 5\ cm, BC = 8\ cm, m\angle CAB = 2(53.13) = 106.26°$, *arc length* $= 9.27\ cm$, *area* $= 23.18\ cm^2$

6. Given circle A with $\angle BAC \cong \angle BAD$, find the following (round to the nearest hundredth, if necessary):

   a. $m\overset{\frown}{CD}$

   $45°$

   b. $m\overset{\frown}{CBD}$

   $315°$

   c. $m\overset{\frown}{BCD}$

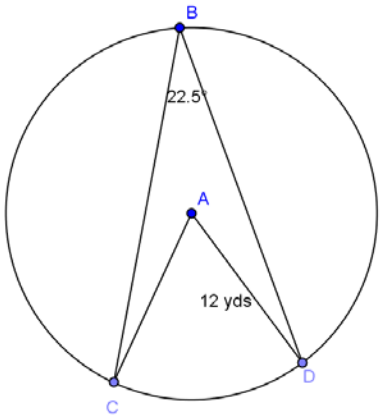
   $202.5°$

   d. Arc length $\overset{\frown}{CD}$

   9.42 *yds*

   e. Arc length $\overset{\frown}{CBD}$

   65.97 *yds*



f. Arc length $\overset{\frown}{BCD}$

42.41 *yds*

g. Area of sector $\overset{\frown}{CD}$

56.55 *yds*^{2}

h. Area of sector $\overset{\frown}{CBD}$

395.84 *yds*^{2}

i. Area of sector $\overset{\frown}{BCD}$

254.47 *yds*^{2}

7. Given circle A, find the following (round to the nearest hundredth, if necessary):

a. Circumference of circle A

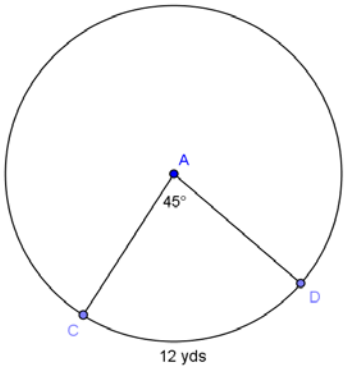
96 *yds*

b. Radius of circle A

15.28 *yds*

c. Area of sector $\overset{\frown}{CD}$

91.69 *yds*^{2}



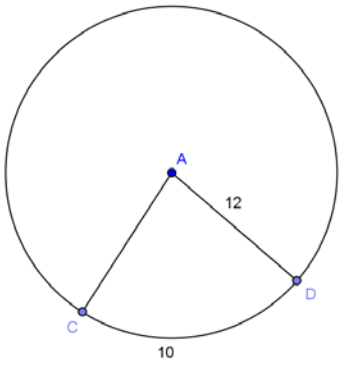
8. Given circle A, find the following (round to the nearest hundredth, if necessary):

a. $m\angle CAD$

$47.74°$

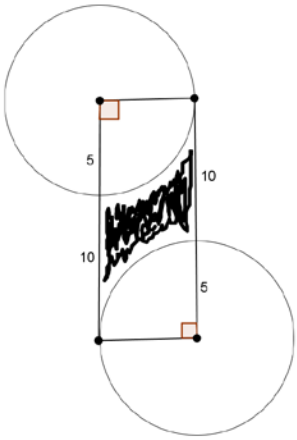
b. Area of sector $\overset{\frown}{CD}$

60



9. **Find the area of the shaded region (round to the nearest hundredth).**

   35.73



10. **Many large cities are building or have built mega Ferris wheels. One is 600 feet in diameter and has 48 cars each seating up to 20 people. Each time the Ferris wheel turns θ degrees, a car is in a position to load.**

    a. **How far does a car move with each rotation of θ degrees (round to the nearest whole number)?**

       39 *feet*

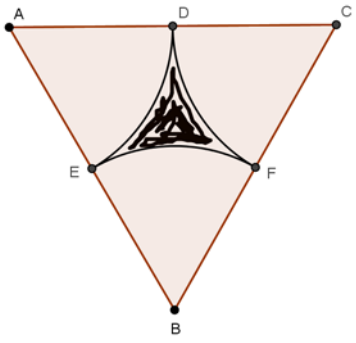
    b. **What is the value of θ in degrees?**

       7.50°

11. **ΔABC is an equilateral triangle with edge length 20 cm. D, E, and F are midpoints of the sides. The vertices of the triangle are the centers of the circles creating the arcs shown. Find the following (round to the nearest hundredth):**

    a. **The area of the sector with center A.**

       $52.36\ cm^2$



    b. **The area of triangle ABC.**

       $173.21\ cm^2$

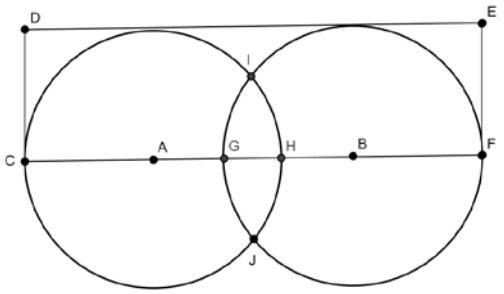
    c. **The area of the shaded region.**

       $16.13\ cm^2$

    d. **The perimeter of the shaded region.**

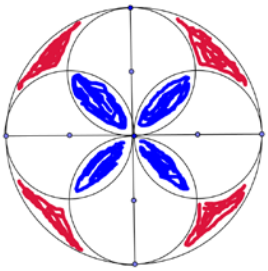
       $31.42\ cm$

12. **In the figure shown, $AC = BF = 5$ cm, $GH = 2$ cm, and $m\angle HBI = 30°$. Find the area in the rectangle, but outside of the circles (round to the nearest hundredth).**



$13.72\ cm^2$

13. **This is a picture of a piece of a mosaic tile. If the radius of each smaller circle is 1 inch, find the area of red section, the white section, and the blue section (round to the nearest hundredth).**



Red $= 2.28\ in^2$, White $= 10.28\ in^2$, Blue $= 2.28\ in^2$

Closing (2 minutes)

Present the questions to the class, and have a discussion, or have students answer individually in writing. Use this as a method of informal assessment.

- Explain how to find the area of a sector of a circle if you know the measure of the arc in degrees.
  - *Find the fraction of the circumference by dividing the measure of the arc in degrees by 360, and then multiply by the circumference $2\pi r$.*
- Explain how to find the arc length of an arc if you know the central angle.
  - *Find the fraction of the area by dividing the measure of the central angle in degrees by 360, then multiply by the circumference πr^2.*

Exit Ticket (5 minutes)

Name _____ Date _____

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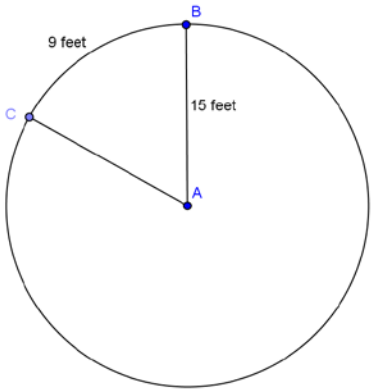
Exit Ticket

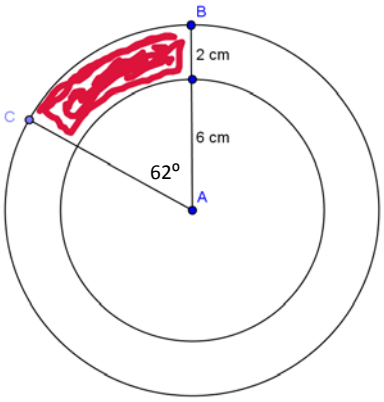
1. Given circle A, find the following (round to the nearest hundredth):



   a. The $m\widehat{BC}$ in degrees.

   b. The area of sector $\widehat{BC}$.

2. Find the shaded area (round to the nearest hundredth).



Exit Ticket Sample Solutions

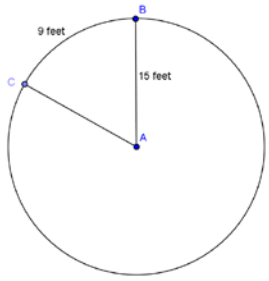
1. Given circle A, find the following (round to the nearest hundredth):

   a. The $m\widehat{BC}$ in degrees.

   $68.75°$

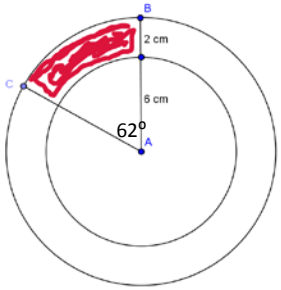
   b. The area of sector $\widehat{BC}$.

   $135\ ft^2$



2. Find the shaded area.

   $15.15\ cm^2$

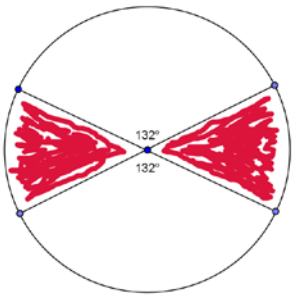


Problem Set Sample Solutions

Students should continue the work they began in class for homework.

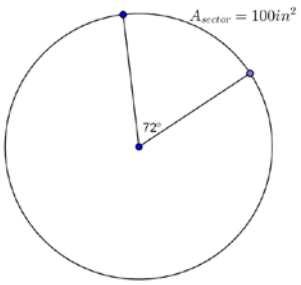
1. Find the area of the shaded region if the diameter is 32 inches (round to the nearest hundredth).

   $214.47\ in^2$

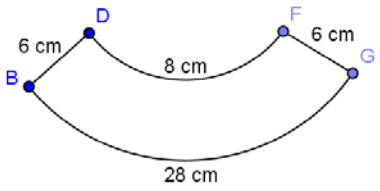


2. Find the area of the entire circle given the area of the sector.

   $500\ in^2$

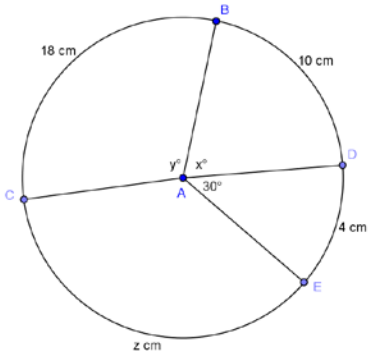


3. $\widehat{DF}$ and $\widehat{BG}$ are arcs of concentric circles with $\overline{BD}$ and $\overline{FG}$ lying on the radii of the larger circle. Find the area of the region (round to the nearest hundredth).



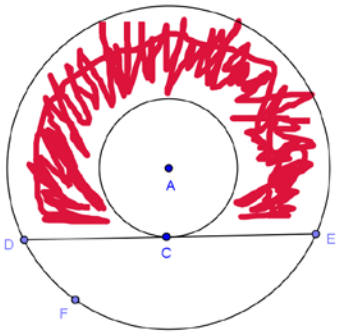
$108.00\ cm^2$

4. Find the radius of the circle, x, y, and z (round to the nearest hundredth).



Radius $= 7.64\ cm$, $x = 74.99°$, $y = 134.99°$, $z = 16\ cm$

5. In the figure, the radii of two concentric circles are 24 cm and 12 cm. $m\widehat{DAE} = 120^0$. If a chord $\overline{DE}$ of the larger circle intersects the smaller circle only at C, find the area of the shaded region in terms of π.



$288\pi\ \text{cm}^2$