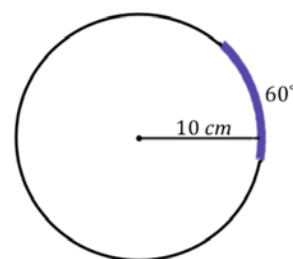


## Lesson 9: Arc Length and Areas of Sectors

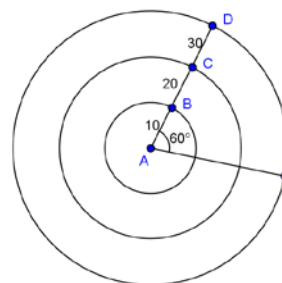
### Classwork

#### Example 1

- a. What is the length of the arc of degree measure  $60^\circ$  in a circle of radius 10 cm?



- b. Given the concentric circles with center  $A$  and with  $m\angle A = 60^\circ$ , calculate the arc length intercepted by  $\angle A$  on each circle. The inner circle has a radius of 10 and each circle has a radius 10 units greater than the previous circle.

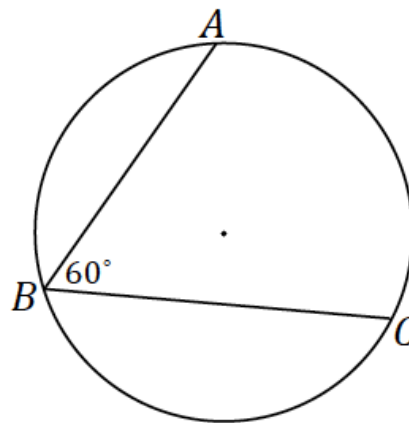


- c. An arc, again of degree measure  $60^\circ$ , has an arc length of  $5\pi$  cm. What is the radius of the circle on which the arc sits?
- d. Give a general formula for the length of an arc of degree measure  $x^\circ$  on a circle of radius  $r$ .

- e. Is the length of an arc intercepted by an angle proportional to the radius? Explain.

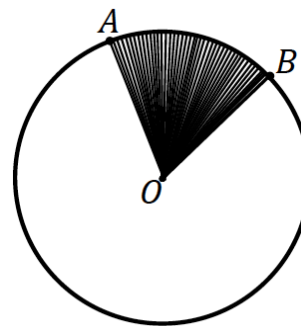
**Exercise 1**

1. The radius of the following circle is 36 cm, and the  $m\angle ABC = 60^\circ$ .
- a. What is the arc length of  $\widehat{AC}$ ?



- b. What is the radian measure of the central angle?

**SECTOR:** Let  $\widehat{AB}$  be an arc of a circle with center  $O$  and radius  $r$ . The union of all segments  $\overline{OP}$ , where  $P$  is any point of  $\widehat{AB}$ , is called a *sector*.

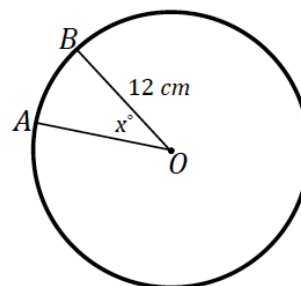


### Example 2

- Circle  $O$  has a radius of 10 cm. What is the area of the circle? Write the formula.
- What is the area of half of the circle? Write and explain the formula.
- What is the area of a quarter of the circle? Write and explain the formula.
- Make a conjecture about how to determine the area of a sector defined by an arc measuring 60 degrees.
- Circle  $O$  has a minor arc  $\widehat{AB}$  with an angle measure of  $60^\circ$ . Sector  $AOB$  has an area of  $24\pi$ . What is the radius of circle  $O$ ?
- Give a general formula for the area of a sector defined by arc of angle measure  $x^\circ$  on a circle of radius  $r$ ?

Exercises 2–3

2. The area of sector  $AOB$  in the following image is  $28\pi$ . Find the measurement of the central angle labeled  $x^\circ$ .

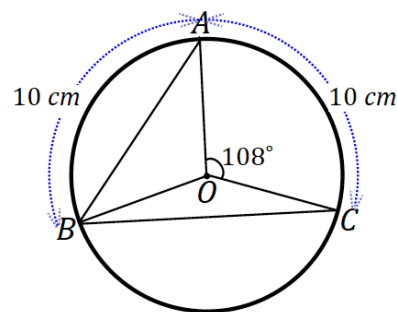


3. In the following figure, circle  $O$  has a radius of 8 cm,  $m\angle AOC = 108^\circ$  and  $\widehat{AB} = \widehat{AC} = 10$  cm. Find:

a.  $\angle OAB$

b.  $\widehat{BC}$

c. Area of sector  $BOC$



## Lesson Summary

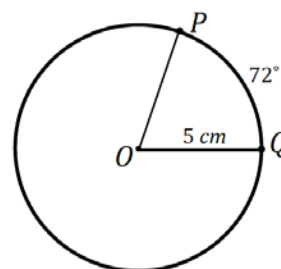
## Relevant Vocabulary

- **ARC:** An *arc* is any of the following three figures—a minor arc, a major arc, or a semicircle.
- **LENGTH OF AN ARC:** The *length of an arc* is the circular distance around the arc.<sup>1</sup>
- **MINOR AND MAJOR ARC:** In a circle with center  $O$ , let  $A$  and  $B$  be different points that lie on the circle but are not the endpoints of a diameter. The *minor arc* between  $A$  and  $B$  is the set containing  $A$ ,  $B$ , and all points of the circle that are in the interior of  $\angle AOB$ . The *major arc* is the set containing  $A$ ,  $B$ , and all points of the circle that lie in the exterior of  $\angle AOB$ .
- **RADIAN:** A *radian* is the measure of the central angle of a sector of a circle with arc length of one radius length.
- **SECTOR:** Let arc  $\widehat{AB}$  be an arc of a circle with center  $O$  and radius  $r$ . The union of the segments  $\overline{OP}$ , where  $P$  is any point on the arc  $\widehat{AB}$ , is called a *sector*. The arc  $\widehat{AB}$  is called the arc of the sector, and  $r$  is called its radius.
- **SEMICIRCLE:** In a circle, let  $A$  and  $B$  be the endpoints of a diameter. A *semicircle* is the set containing  $A$ ,  $B$ , and all points of the circle that lie in a given half-plane of the line determined by the diameter.

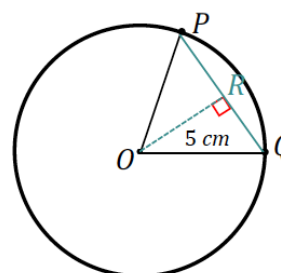
## Problem Set

1.  $P$  and  $Q$  are points on the circle of radius 5 cm and the measure of arc  $\widehat{PQ}$  is  $72^\circ$ . Find, to one decimal place each of the following:

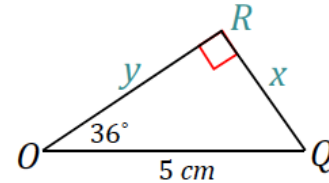
- The length of arc  $\widehat{PQ}$
- Find the ratio of the arc length to the radius of the circle.



- The length of chord  $PQ$



- d. The distance of the chord  $PQ$  from the center of the circle.

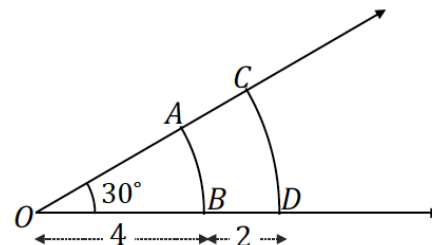


- e. The perimeter of sector  $POQ$ .  
f. The area of the wedge between the chord  $PQ$  and the arc  $\widehat{PQ}$   
g. The perimeter of this wedge.

2. What is the radius of a circle if the length of a  $45^\circ$  arc is  $9\pi$ ?

3. Arcs  $\widehat{AB}$  and  $\widehat{CD}$  both have an angle measure of  $30^\circ$ , but their arc lengths are not the same.  $\overline{OB} = 4$  and  $\overline{BD} = 2$ .

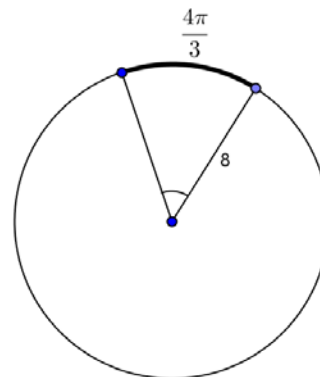
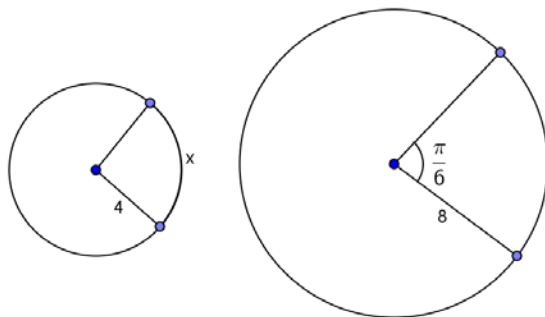
- a. What are the arc lengths of arcs  $\widehat{AB}$  and  $\widehat{CD}$ ?  
b. What is the ratio of the arc length to the radius for all of these arcs? Explain.  
c. What are the areas of the sectors  $AOB$  and  $COD$ ?



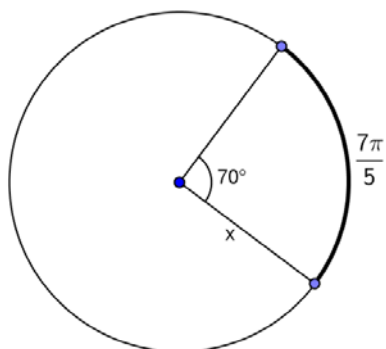
4. In the circles shown, find the value of  $x$ .

The circles shown have central angles that are equal in measure.

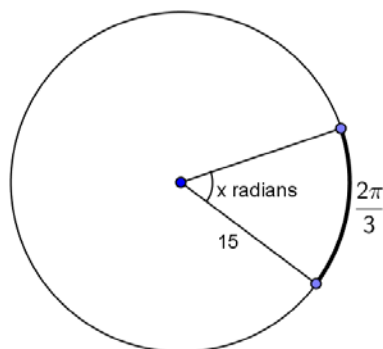
- a. b.



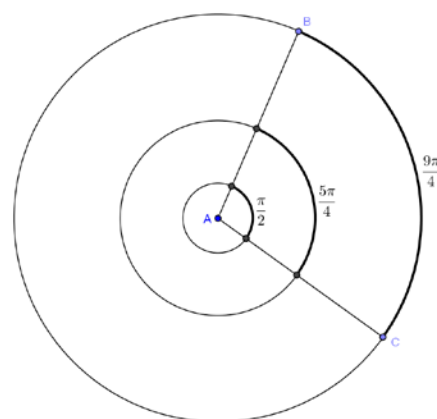
c.



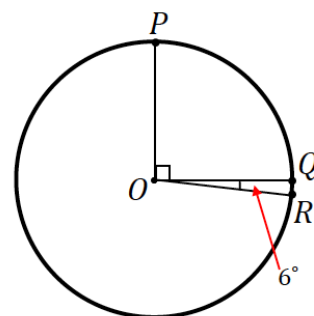
d.



5. The concentric circles all have center  $A$ . The measure of the central angle is  $45^\circ$ . The arc lengths are given.
- Find the radius of each circle.
  - Determine the ratio of the arc length to the radius of each circle, and interpret its meaning.

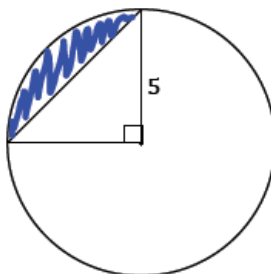


6. In the figure, if  $\widehat{PQ} = 10$  cm, find the length of arc  $\widehat{QR}$ .

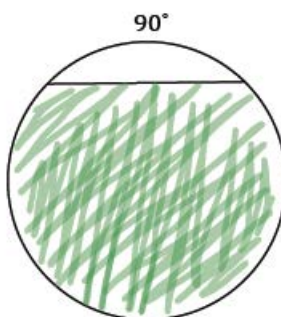


7. Find, to one decimal place, the areas of the shaded regions.

a.



b. The following circle has a radius of 2.



c.

