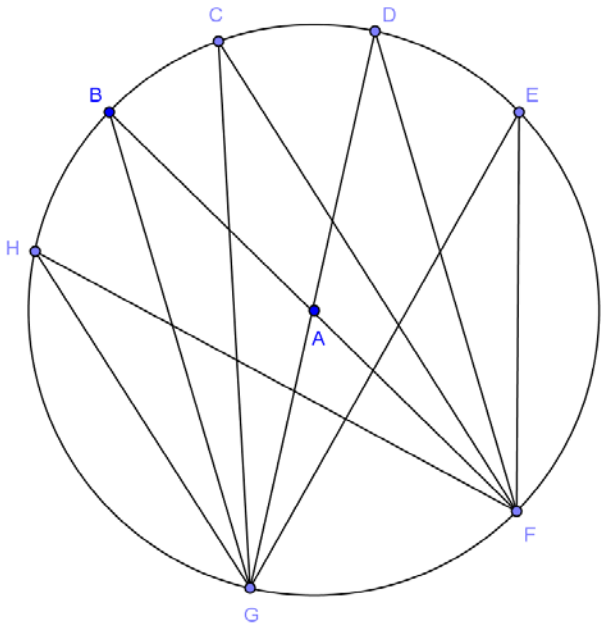
Lesson 7: The Angle Measure of an Arc

Classwork

Opening Exercise

If the measure of $\angle GBF$ is 17°, name 3 other angles that have the same measure and explain why.

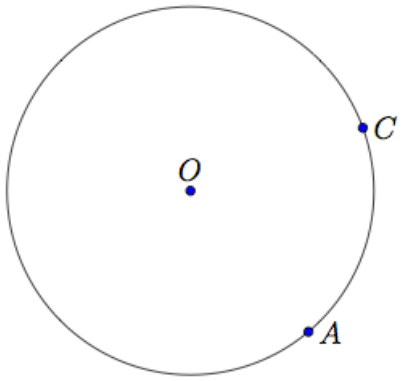


What is the measure of $\angle GAF$? Explain.

Can you find the measure of $\angle BAD$? Explain.

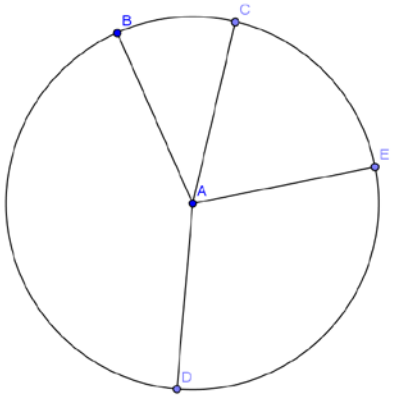
Example 1

What if we started with an angle inscribed in the minor arc between A and C?



Exercises 1–4

1. In circle A, $\widehat{BC}:\widehat{CE}:\widehat{ED}:\widehat{DB} = 1:2:3:4$. Find



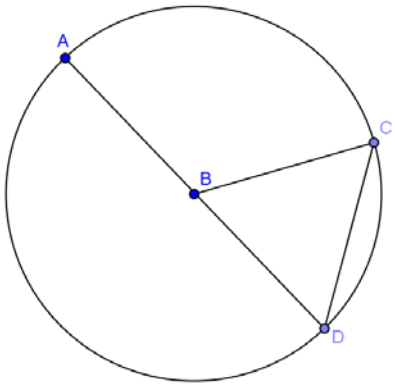
   a. $m\angle BAC$

   b. $m\angle DAE$

   c. $m\widehat{DB}$

   d. $m\widehat{CED}$

2. In circle B, $AB = CD$. Find

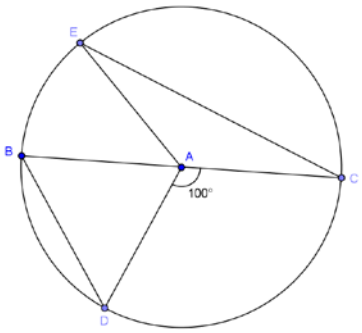


a. $m\widehat{CD}$

b. $m\widehat{CAD}$

c. $m\widehat{AD}$

3. In circle A, $\overline{BC}$ is a diameter and $m\angle DAC = 100^0$. If $m\widehat{EC} = 2m\widehat{BD}$, find

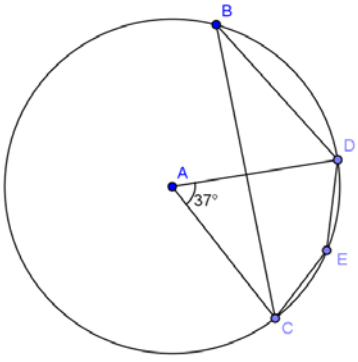


a. $m\angle BAE$

b. $m\widehat{EC}$

c. $m\widehat{DEC}$

4. Given circle A with $m\angle CAD = 37^0$, find



a. $m\widehat{CBD}$

b. $m\angle CBD$

c. $m\angle CED$

Lesson Summary

THEOREMS:

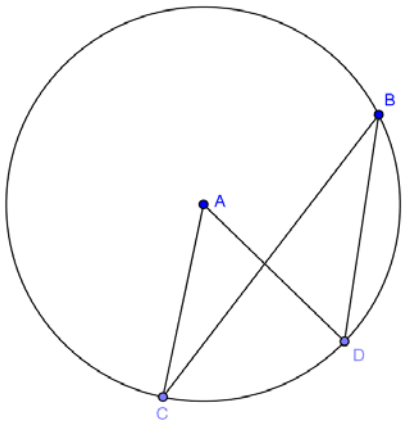
- **INSCRIBED ANGLE THEOREM**: The measure of an inscribed angle is half the measure of its intercepted arc.
- Two arcs (of possibly different circles) are similar if they have the same angle measure. Two arcs in the same or congruent circles are congruent if they have the same angle measure.
- All circles are similar.

Relevant Vocabulary

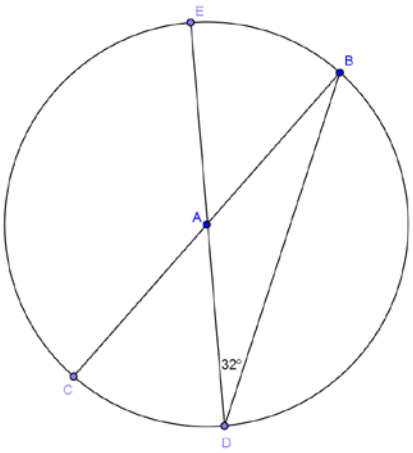
- **ARC**: An *arc* is a portion of the circumference of a circle.
- **MINOR AND MAJOR ARC:** Let C be a circle with center O, and let A and B be different points that lie on C but are not the endpoints of the same diameter. The *minor arc* is the set containing A, B, and all points of C that are in the interior of $\angle AOB$. The *major arc* is the set containing A, B, and all points of C that lie in the exterior of $\angle AOB$.
- **SEMICIRCLE**: In a circle, let A and B be the endpoints of a diameter. A *semicircle* is the set containing A, B, and all points of the circle that lie in a given half-plane of the line determined by the diameter.
- **INSCRIBED ANGLE**: An *inscribed angle* is an angle whose vertex is on a circle and each side of the angle intersects the circle in another point.
- **CENTRAL ANGLE**: A *central angle* of a circle is an angle whose vertex is the center of a circle.
- **INTERCEPTED ARC OF AN ANGLE**: An angle *intercepts* an arc if the endpoints of the arc lie on the angle, all other points of the arc are in the interior of the angle, and each side of the angle contains an endpoint of the arc.

Problem Set

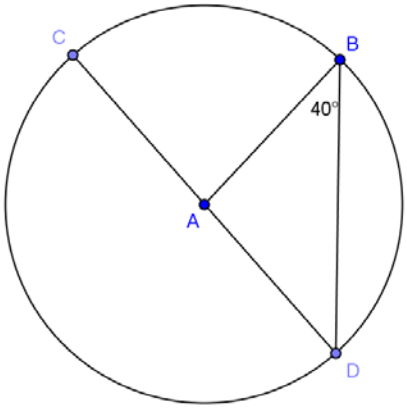
1. Given circle A with $m\angle CAD = 50^0$,
   a. Name a central angle.
   b. Name an inscribed angle.
   c. Name a chord.
   d. Name a minor arc.
   e. Name a major arc.
   f. Find $m\widehat{CD}$,
   g. Find $m\widehat{CBD}$.
   h. Find $m\angle CBD$.



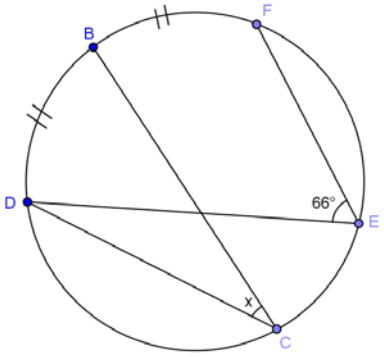
2. Given circle A, find the measure of each minor arc.



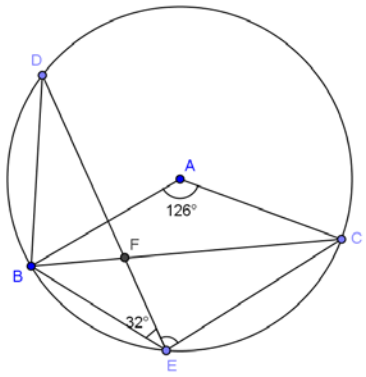
3. Given circle A, find
   a. $m\angle BAD$
   b. $m\angle CAB$
   c. $m\widehat{BC}$
   d. $m\widehat{BD}$
   e. $m\widehat{BCD}$



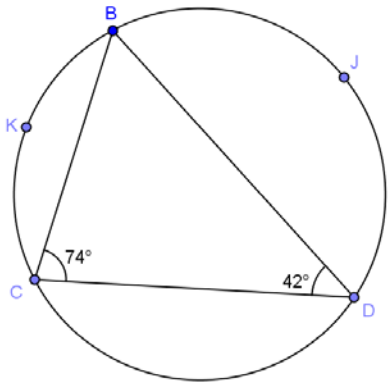
4. Find the angle measure of angle x.



5. In the figure, $m\angle BAC = 126^0$ and $m\angle BED = 32^0$. Find $m\angle DEC$.



6. In the figure $m\angle BCD = 74^0$, and $m\angle BDC = 42^0$. K is the midpoint of $\overset{\frown}{CB}$ and J is the midpoint of $\overset{\frown}{BD}$. Find $m\angle KBD$ and $m\angle CKJ$.



Solution: Join BK, KC, KD, KJ, JC, and JD.

$\overset{\frown}{mBK} = m\overset{\frown}{KC}$ _____

$m\angle KDC = \frac{42}{2} = 21^0$ _____

a = _____ _____

In ΔBCD, b = _____ _____

c = _____ _____

$\overset{\frown}{mBJ} = m\overset{\frown}{JD}$ _____

$m\angle JCD$ = _____ _____

d = _____ _____

$m\angle KBD = a + b =$ _____

$m\angle CKJ = c + d =$ _____