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Lesson 2: Circles, Chords, Diameters, and Their Relationships

Student Outcomes

* Identify the relationships between the diameters of a circle and other chords of the circle.

Lesson Notes

Students are asked to construct the perpendicular bisector of a line segment and draw conclusions about points on that bisector and the endpoints of the segment. They relate the construction to the theorem stating that any perpendicular bisector of a chord must pass through the center of the circle.

Classwork

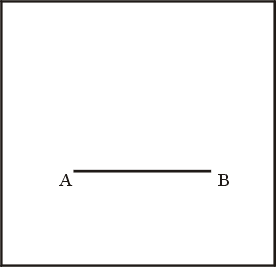
Opening Exercise (4 minutes)

*Scaffolding:*

Post a diagram and display the steps to create a perpendicular bisector used in Lesson 4 of Module 1.

* Label the endpoints of the segment and .
* Draw circle with center and radius .
* Draw circle with center and radius .
* Label the points of intersection as and .
* Draw .

Opening Exercise

Construct the perpendicular bisector of line segment below (as you did in Module 1).

Draw another line that bisects but is not perpendicular to it.

List one similarity and one difference between the two bisectors.

Answers will vary. **All points on the perpendicular bisector are equidistant from points and** . Points on the other bisector are not equidistant from points A and B. The perpendicular bisector meets at right angles. The other bisector meets at angles that are not congruent.

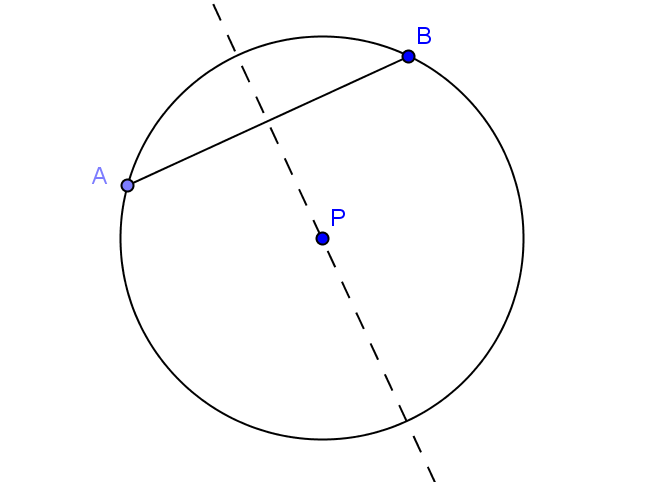
You may wish to recall for students the definition of *equidistant*:

**Equidistant:**. A point is said to be *equidistant* from two different points and if .

Points and can be replaced in the definition above with other figures (lines, etc.) as long as the distance to those figures is given meaning first. In this lesson, we will define the distance from the center of a circle to a chord. This definition will allow us to talk about the center of a circle as being equidistant from two chords.

**Discussion (12 minutes)**

Ask students independently or in groups to each draw chords and describe what they notice. Answers will vary depending on what each student drew.

Lead students to relate the perpendicular bisector of a line segment to the points on a circle, guiding them toward seeing the relationship between the perpendicular bisector of a chord and the center of a circle.

* Construct a circle of any radius, and identify the center as point .
* Draw a chord, and label it .
* Construct the perpendicular bisector of .
* What do you notice about the perpendicular bisector of ?
  + *It passes through point P, the center of the circle.*
* Draw another chord and label it .
* Construct the perpendicular bisector of .
* What do you notice about the perpendicular bisector of ?
  + *It passes through point , the center of the circle.*
* What can you say about the points on a circle in relation to the center of the circle?
  + *The center of the circle is equidistant from any two points on the circle.*

**MP.3**

**&**

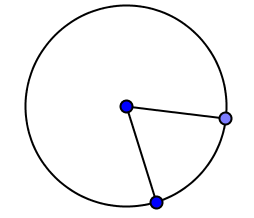
**MP.7**

* Look at the circles, chords, and perpendicular bisectors created by your neighbors. What statement can you make about the perpendicular bisector of any chord of a circle? Why?
  + *It must contain the center of the circle. The center of the circle is equidistant from the two endpoints of the chord because they lie on the circle. Therefore, the center lies on the perpendicular bisector of the chord. That is, the perpendicular bisector contains the center.*
* How does this relate to the definition of the perpendicular bisector of a line segment?

*Scaffolding:*

* Review the definition of *central angle* by posting a visual guide.
* A central angle of a circle is an angle whose vertex is the center of a circle.

* + *The set of all points equidistant from two given points (endpoints of a line segment) is precisely the set of all points on the perpendicular bisector of the line segment.*



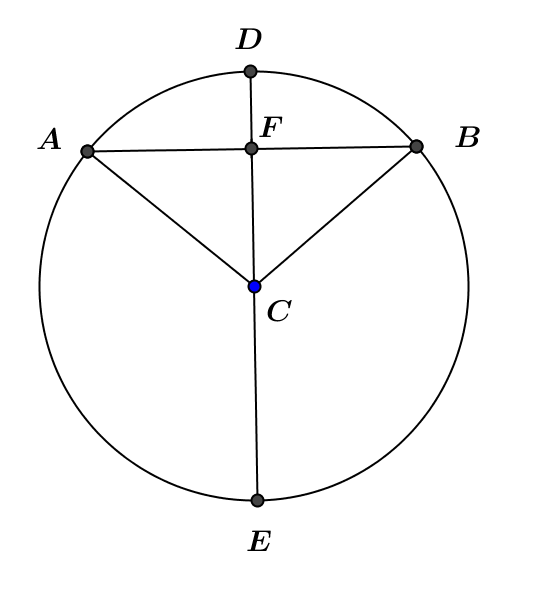
Exercises 1–6 (20 minutes)

Assign one proof to each group, and then jigsaw, share, and gallery walk as students present their work.

Exercises 1–6

1. Prove the theorem: *If a diameter of a circle bisects a chord, then it must be perpendicular to the chord.*

Draw a diagram similar to that shown below.



Given: Circle with diameter , chord , and .

Prove:

Given

Reflexive property

Radii of the same circle are equal in measure

SSS

Corresponding angles of congruent triangles are equal in measure

and are right angles Equal angles that form a linear pair each measure

Definition of perpendicular lines

OR

Given

Radii of the same circle are equal in measure

Base angles of an isosceles are equal in measure

SAS

Corresponding angles of congruent triangles are equal in measure

and are right angles Equal angles that form a linear pair each measure

Definition of perpendicular lines

1. Prove the theorem: *If a diameter of a circle is perpendicular to a chord, then it bisects the chord.*

Use a diagram similar to that in Exercise 1 above.

Given: Circle with diameter , chord , and

Prove: bisects

Given

and are right angles Definition of perpendicular lines

and are right triangles Definition of right triangle

All right angles are congruent

Reflexive property

Radii of the same circle are equal in measure

HL

Corresponding sides of congruent triangles are equal in length

bisects Definition of segment bisector

OR

Given

and are right angles Definition of perpendicular lines

All right angles are congruent

Radii of the same circle are equal in measure

Base angles of an isosceles triangle are congruent

Two angles of triangle are equal in measure, so third angles are equal

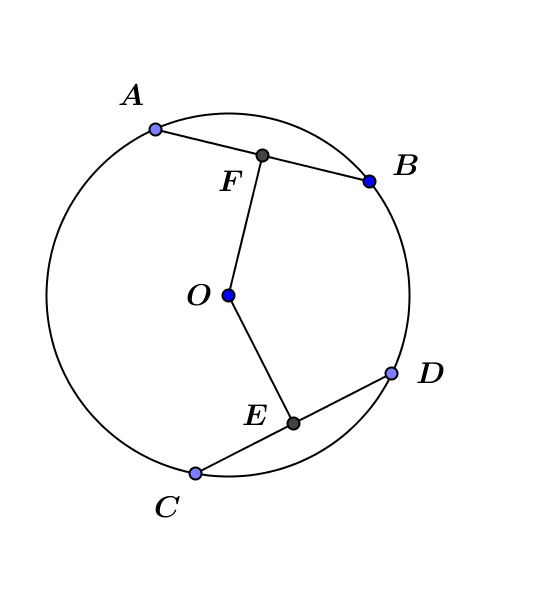
ASA

Corresponding sides of congruent triangles are equal in length

bisects Definition of segment bisector

1. The distance from the center of a circle to a chord is defined as the length of the perpendicular segment from the center to the chord. Note that, since this perpendicular segment may be extended to create a diameter of the circle, therefore, the segment also bisects the chord, as proved in Exercise 2 above.

Prove the theorem: *In a circle, if two chords are congruent, then the center is equidistant from the two chords.*

Use the diagram below.

Given: Circle with chords and ; ; is the midpoint of and is the midpoint of .

Prove:

Given

; If a diameter of a circle bisects a chord, then the diameter must be perpendicular to the chord.

and are right angles Definition of perpendicular lines

and are right triangles Definition of right triangle

E and are midpoints of and Given

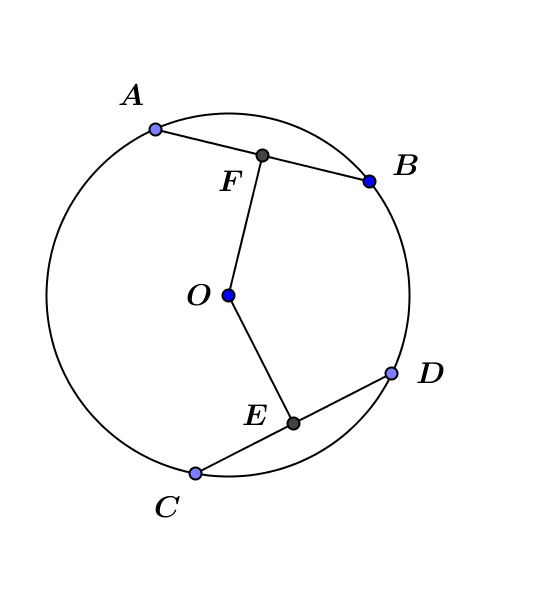
and and are midpoints of and

All radii of a circle are equal in measure

HL

Corresponding sides of congruent triangles are equal in length

1. Prove the theorem: *In a circle, if the center is equidistant from two chords, then the two chords are congruent.*

Use the diagram below.

Given: Circle with chords and ; ; is the midpoint of and is the midpoint of

Prove:

Given

; If a diameter of a circle bisects a chord, then it must be perpendicular to the chord.

and are right angles Definition of perpendicular lines.

and are right triangles Definition of right triangle.

All radii of a circle are equal in measure.

HL

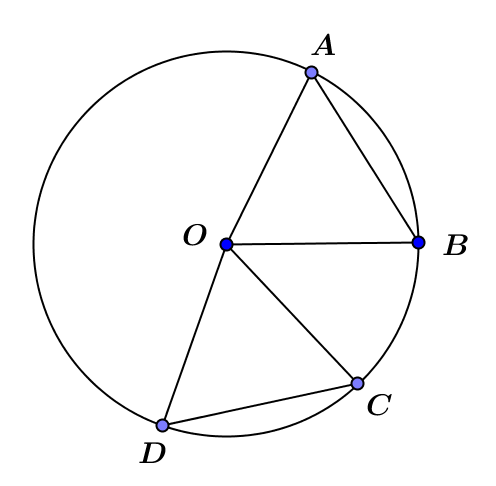
Corresponding sides of congruent triangles are equal in length.

is the midpoint of , and is the   
midpoint of Given

and and are midpoints of and .

1. A central angle defined by a chord is an angle whose vertex is the center of the circle and whose rays intersect the circle. The points at which the angle’s rays intersect the circle form the endpoints of the chord defined by the central angle.  
     
   Prove the theorem: *In a circle, congruent chords define central angles equal in measure.*

Use the diagram below.



We are given that the two chords ( and ) are congruent. Since all radii of a circle are congruent, . Therefore, by SSS. since corresponding angles of congruent triangles are equal in measure.

1. Prove the theorem: In a circle, if two chords define central angles equal in measure, then they are congruent.

Using the diagram from Exercise 5 above, we now are given that . Since all radii of a circle are congruent, . Therefore, by SAS. because corresponding sides of congruent triangles are congruent

Closing (4 minutes)

Have students write all they know to be true about the diagrams below. Bring the class together, go through the Lesson Summary, having students complete the list that they started, and discuss each point.

A reproducible version of the graphic organizer shown is included at the end of the lesson.

|  |  |  |
| --- | --- | --- |
| Diagram | Explanation of Diagram | Theorem or Relationship |
|  | Diameter of a circle bisecting a chord | If a diameter of a circle bisects a chord, then it must be perpendicular to the chord.  If a diameter of a circle is perpendicular to a chord, then it bisects the chord. |
|  | Two congruent chords equidistance from center | If two chords are congruent, then the center is equidistant from the two chords.  If the center is equidistant from two chords, then the two chords are congruent. |
|  | Congruent chords | Congruent chords define central angles equal in measure.  If two chords define central angles equal in measure, then they are congruent. |

Lesson Summary

Theorems about chords and diameters in a circle and their converses

* **If a diameter of a circle bisects a chord, then it must be perpendicular to the chord.**
* **If a diameter of a circle is perpendicular to a chord, then it bisects the chord.**
* **If two chords are congruent, then the center is equidistant from the two chords.**
* **If the center is equidistant from two chords, then the two chords are congruent.**
* **Congruent chords define central angles equal in measure.**
* **If two chords define central angles equal in measure, then they are congruent.**

**Relevant Vocabulary**

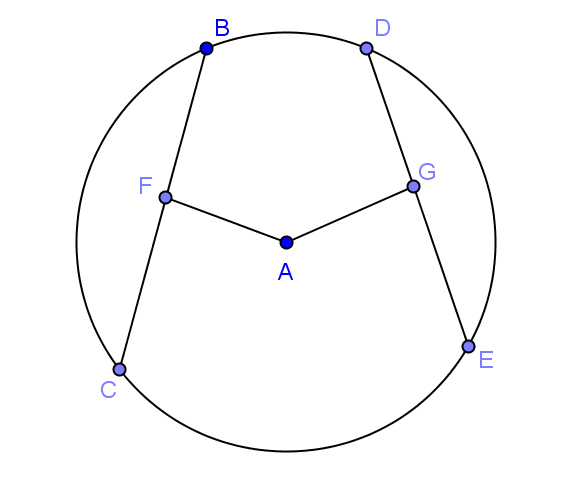
**Equidistant: A point is said to be *equidistant* from two different points and if .**

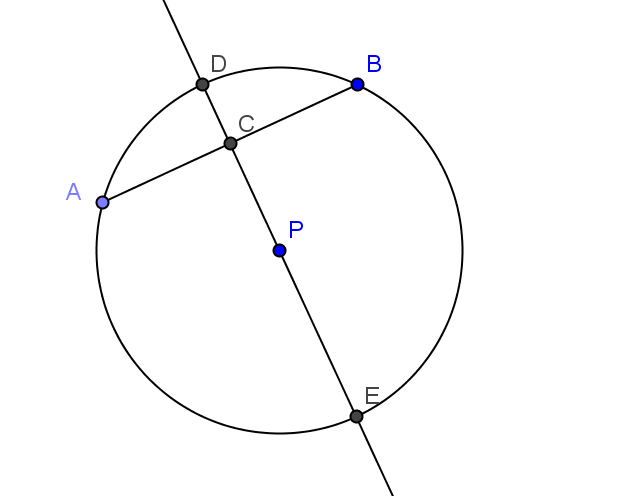
Exit Ticket (5 minutes)

Name Date

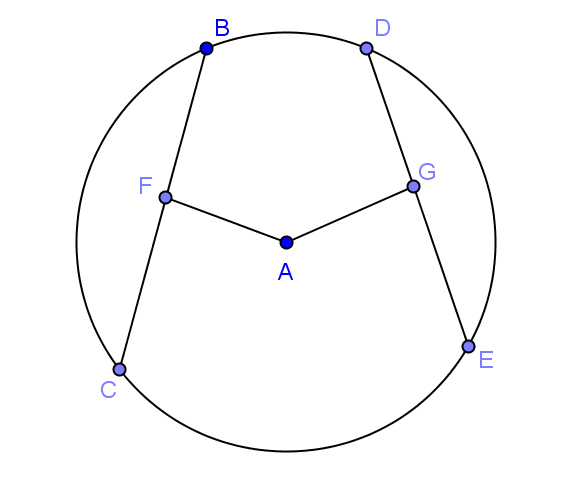
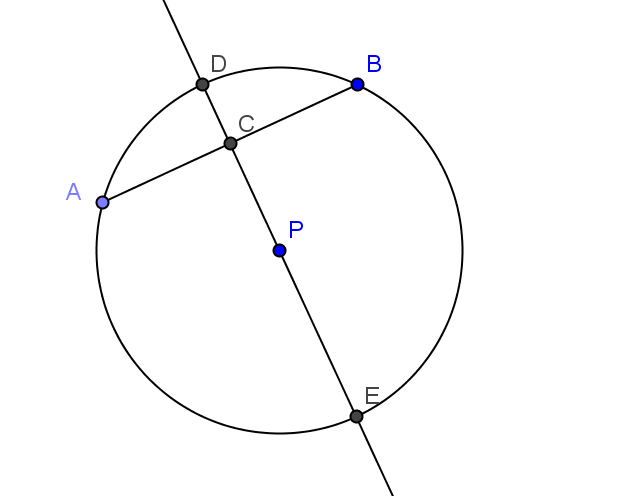
Lesson 2: Circles, Chords, Diameters, and Their Relationships

Exit Ticket

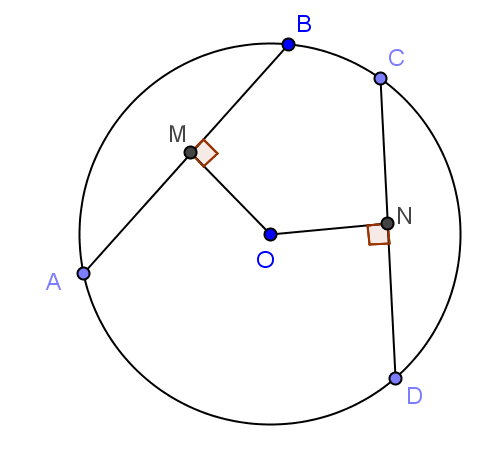


1. Given circle shown, and . Find .
2. In the figure, circle has a radius of . .
   1. If , what is the length of ?
   2. If , what is the length of ?

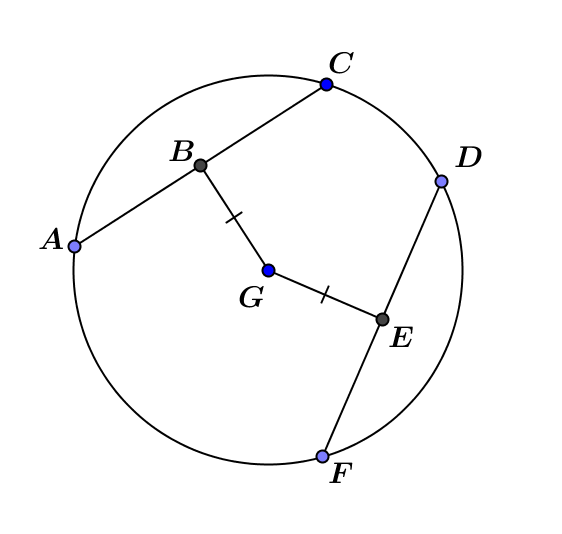
Exit Ticket Sample Solutions

1. Given circle shown and . Find .
2. In the figure, circle has a radius of . .
3. If , what is the length of ?
4. If , what is the length of ?

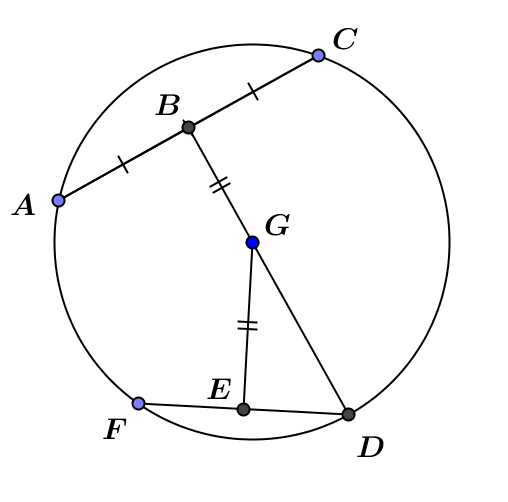
Problem Set Sample Solutions

1. In this drawing, ,, and . What is ?

()

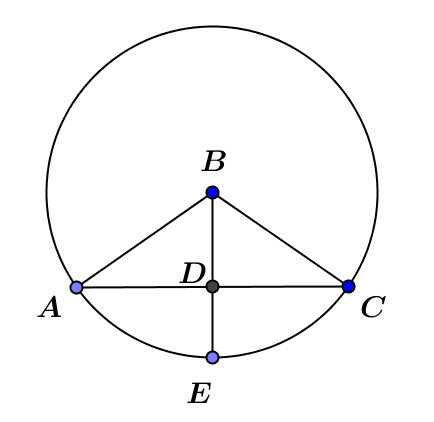
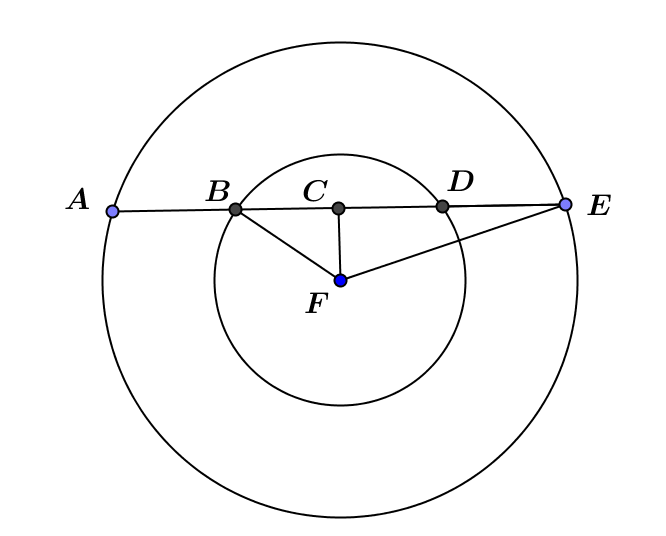
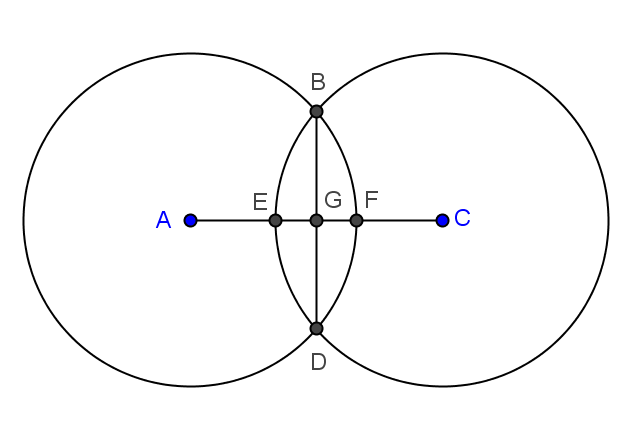


1. In the figure to the right, and ; . Find .



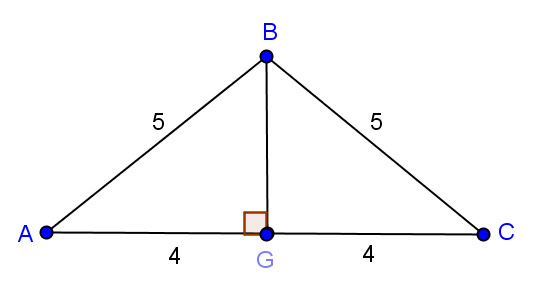
1. In the figure, , and. Find . Explain your work.

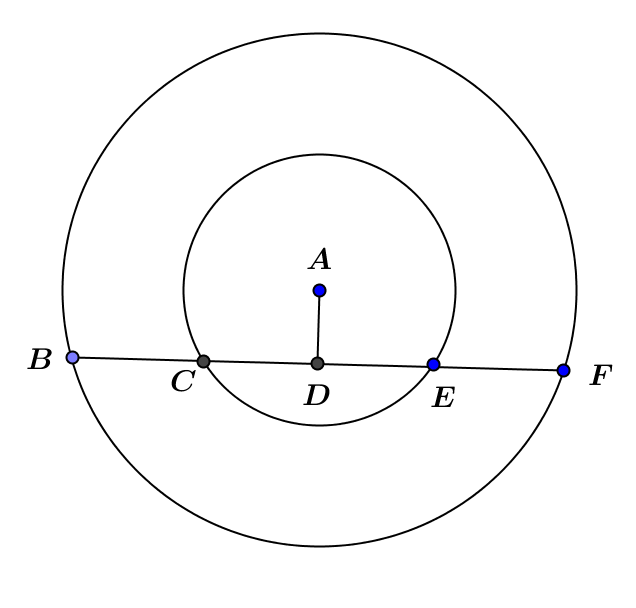
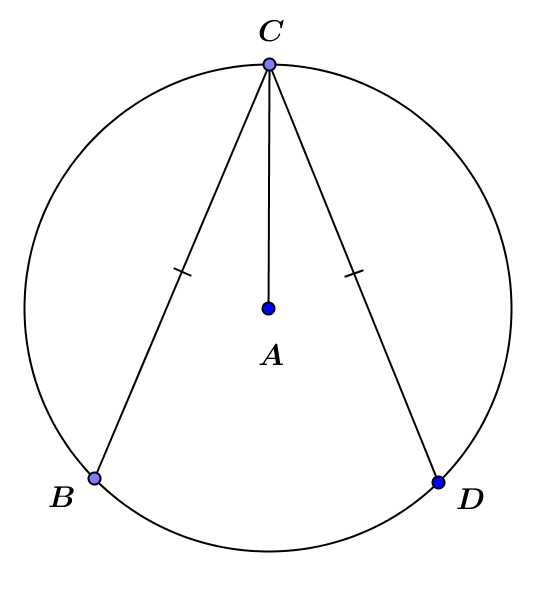
*, is a right triangle with and , so by Pythagorean theorem. .*

1. In the figure, . Find .
2. In the figure, and the two concentric circles have radii of and .   
   Find .
3. In the figure, the two circles have equal radii and intersect at points and . and are centers of the circles. , and the radius of each circle is . . Find . Explain your work.

(radii)

(Pythagorean theorem)



1. In the figure, the two concentric circles have radii of and . Chord of the larger circle intersects the smaller circle at and . . .
   1. Find .
   2. Find .
2. In the figure, is the center of the circle, and . Prove that bisects *.*

Let and be perpendiculars from to and , respectively.

Given

If two chords are congruent, then the center is equidistant from the two chords.

Reflexive property

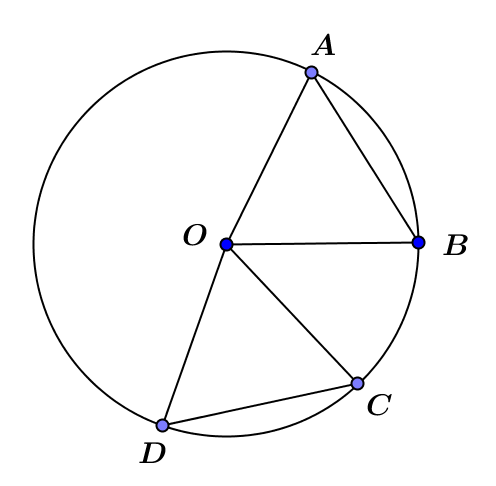
A line joining the center and the midpoint of a chord is perpendicular to the chord.

HL

Corresponding angles of congruent triangles are equal in measure.

bisects Definition of angle bisector

1. In class, we proved: *Congruent chords define central angles equal in measure*.
   1. Give another proof of this theorem based on the properties of rotations. Use the figure from Exercise 5.



We are given that the two chords ( and ) are congruent. Therefore, a rigid motion exists that carries to . The same rotation that carries to also carries to and to . The angle of rotation is the measure of , and the rotation is clockwise.

* 1. Give a rotation proof of the converse: *If two chords define central angles of the same measure, then they must be congruent.*

Using the same diagram, we are given that . Therefore, a rigid motion (a rotation) carries to . This same rotation carries to and to . The angle of rotation is the measure of , and the rotation is clockwise.

Graphic Organizer on Circles

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| **Diagram** | **Explanation of Diagram** | **Theorem or Relationship** |
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