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Probability and Statistics

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Precalculus • Module 5

Probability and Statistics

OVERVIEW

In this module, students build on their understanding of probability developed in previous grades. In Topic A, the multiplication rule for independent events introduced in Grade 11 is generalized to a rule that can be used to calculate the probability of the intersection of two events in situations where the two events are not independent. In this topic, students are also introduced to three techniques for counting outcomes—the fundamental counting principle, permutations, and combinations. These techniques are then used to calculate probabilities, and these probabilities are interpreted in context (**S-CP.B.8**, **S-CP.B.9**).

In Topic B, students study probability distributions for discrete random variables (**S-MD.A.1**). They develop an understanding of the information that a probability distribution provides and interpret probabilities from the probability distribution of a discrete random variable in context (**S-MD.A.2**). For situations where the probabilities associated with a discrete random variable can be calculated given a description of the random variable, students determine the probability distribution (**S-MD.A.3**). Students also see how empirical data can be used to approximate the probability distribution of a discrete random variable (**S-MD.A.4**). This topic also introduces the idea of expected value, and students calculate and interpret the expected value of discrete random variables in context.

Topic C is a capstone topic for this module, where students use what they have learned about probability and expected value to analyze strategies and make decisions in a variety of contexts (**S-MD.B.5**, **S-MD.B.6**, **S-MD.B.7**). Students use probabilities to make a fair decision and explain how to make fair and "unfair" decisions. Students analyze simple games of chance as they calculate and interpret the expected payoff in context. They make decisions based on expected values in problems with business, medical, and other contexts. They also examine and interpret what it means for a game to be fair. Interpretation and explanations of expected values are important outcomes for Topic C.

Focus Standards

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

S-CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Calculate expected values and use them to solve problems.

S-MD.A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S-MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

S-MD.A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

S-MD.A.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in* 100 *randomly selected households?*

Use probability to evaluate outcomes of decisions.

S-MD.B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

S-MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S-MD.B.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Foundational Standards

Understand independence and conditional probability and use them to interpret data.

S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S-CP.A.3 Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S-CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) – P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively**. Students interpret probabilities calculated using the addition rule and the general multiplication rule. They use permutations and combinations to calculate probabilities and interpret them in context. Students also explain the meaning of the expected value of a random variable as a long-run average and connect this interpretation to the given context.

MP.3 **Construct viable arguments and critique the reasoning of others**. Students construct arguments in distinguishing between situations involving combinations and those involving permutations. Students use permutations and combinations to calculate probabilities and evaluate decisions based on probabilities. Students also use expected values to analyze games of chance and to evaluate whether a game is "fair." Students design, compare, and evaluate games of chance that they construct, comparing their games to the games of other students based on probabilities and expected values. They analyze strategies based on probability. For example, students use expected value to explain which of two plans yields the largest earnings for an insurance company.

MP.4 **Model with mathematics**. Students develop a probability distribution for a random variable by finding the theoretical probabilities. Students model probability distributions by estimating probabilities empirically. They use probabilities to make and justify decisions. Throughout the module, students use statistical ideas to explain and solve real-world problems. For example, given the probability of finding a female egg in a nest, students determine a discrete probability distribution for the number of male eggs in the nest.

MP.5 **Use appropriate tools strategically**. Students use technology to carry out simulations in order to estimate probabilities empirically. For example, students use technology to simulate a dice-tossing game and generate random numbers to simulate the flavors in a pack of cough drops. They use technology to graph a probability distribution and to calculate expected values. Students come to view discrete probability distributions as tools that can be used to understand real-world situations and solve problems.

MP.8 **Look for and express regularity in repeated reasoning**. Students use simulations to observe the long-run behavior of a random variable, using the results of the simulations to estimate probabilities.

Terminology

New or Recently Introduced Terms

- **Combination of k items selected from a set of n distinct items** (an unordered set of k items selected from a set of n distinct items).
- **Continuous random variables** (a random variable for which the possible values form an entire interval along the number line).
- **Discrete random variables** (a random value for which the possible values are isolated points along the number line).
- **Empirical probability** (a probability that has been estimated by observing a large number of outcomes of a chancge experiment or values of a random variable).
- **Expected value of a random variable** (the long-run average value expected over a large number of observations of the value of a random variable).
- **Fundamental counting principle** (Let n_1 be the number of ways the first step or event can occur and n_2 be the number of ways the second step or event can occur. Continuing in this way, let n_k be the number of ways the k^{th} stage or event can occur. Then, based on the fundamental counting principle, the total number of different ways the process can occur is $n_1 * n_2 * n_3 * \ldots . * n_k$.)
- **General multiplication rule** (a probability rule for calculating the probability of the intersection of two events).
- **Long-run behavior of a random variable** (the behavior of the random variable over a very long sequence of observations).
- **Permutation of k items selected from a set of n distinct items** (an ordered sequence of k items selected from a set of n distinct items).
- **Probability distribution** (a table or graph that provides information about the long-run behavior of a random variable).
- **Probability distribution of a discrete random variable** (a table or graph that specifies the possible values of the random variable and the associated probabilities).
- **Random variable** (a variable whose possible values are based on the outcome of a random event).

- **Theoretical probability** (a probability calculated by assigning a probability to all possible outcomes in the sample space for a chance experiment).
- **Uniform probability model** (a probability distribution that assigns equal probability to each possible outcome of a chance experiment).

Familiar Terms and Symbols[2]

- Chance experiment
- Complement of an event
- Event
- Intersection of events
- Sample space
- Union of events

Suggested Tools and Representations

- Graphing calculator or graphing software
- Random number software
- Random number tables
- Two-way frequency tables

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2, S-MD.A.3, S-MD.A.4
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	S-CP.B.8, S-CP.B.9, S-MD.A.2, S-MD.A.3, S-MD.B.5, S-MD.B.6, S-MD.B.7

[2] These are terms and symbols students have seen previously.