Name ______________________________ Date ________________

1. In the game of tennis, one player serves to a second player to start a point. If the server misses landing the first serve (the ball is hit out of bounds and cannot be played), the server is allowed a second serve. Suppose a particular tennis player, Chris, has a 0.62 probability of a first serve landing in bounds and a 0.80 probability of a second serve landing in bounds. Once a serve has landed in bounds, the players take turns hitting the ball back and forth until one player hits the ball out of bounds or into the net. The player who hits the ball out or into the net loses the point, and the other player wins the point. For Chris, when the first serve lands in bounds, he has a 75% chance of winning the point; however, he has only a 22% chance of winning the point on his second serve.

   a. Calculate the probability that Chris lands his first serve in bounds and then goes on to win the point for a randomly selected point.

   b. Calculate the probability that Chris misses the first serve, lands the second serve, and then wins the point.

   c. Calculate the probability that Chris wins a randomly selected point.

2. In Australia, there are approximately 100 species of venomous snakes, but only about 12 have a deadly bite. Suppose a zoo randomly selects one snake from each of five different species to show visitors. What is the probability that exactly one of the five snakes shown is deadly?

3. In California (CA), standard license plates are currently of the form: 1 number–3 letters–3 numbers. Assume the numbers are $0–9$ and the letters are A–Z.

   a. In theory, how many different possible standard CA license plates are there, assuming we can repeat letters and numbers?

   b. How many different possible standard CA license plates are there if we are not allowed to repeat any letters or numbers?

   c. For part (b), did you use permutations or combinations to carry out the calculation? Explain how you know.

4. When there is a problem with a computer program, many people call a technical support center for help. Several people may call at once, so the center needs to be able to have several telephone lines available at the same time. Suppose we want to consider a random variable that is the number of telephone lines in use by the technical support center of a software manufacturer at a particular time of day. Suppose that the probability distribution of this random variable is given by the following table:

x	0	1	2	3	4	5
$p(x)$	0.35	0.20	0.15	0.15	0.10	0.05

a. Produce a graph of the probability distribution for this random variable, including all relevant labels.

b. Calculate the expected value of the random variable.

c. Explain how to interpret this expected value in context.

5. The following table lists the number of U.S. households (in thousands) with 0 vehicles, 1 vehicle, 2 vehicles, or 3 or more vehicles for all households responding to the 2009 National Household Travel Survey.

No vehicle	One vehicle	Two vehicles	Three or more vehicles
$9{,}828$	$36{,}509$	$41{,}077$	$25{,}668$

a. Use these data to create a table of relative frequencies that could be used as estimates of the probability distribution of number of vehicles for a randomly selected U.S. household (that responded to the survey).

No vehicle	One vehicle	Two vehicles	Three or more vehicles

b. Suppose you want to examine the distribution of the number of vehicles in all U.S. households. Define a random variable that corresponds to the probability distribution in part (a).

c. Assume for the moment that the last column corresponds to exactly three vehicles. Calculate the expected number of vehicles per household.

d. Now reconsider the last category. Suppose we were to find the information for the actual number of vehicles for these households. Would the expected number of vehicles per household with this new information be larger or smaller than the expected value you found in (c)? Explain your reasoning.

e. Suppose a town has about 4,500 households. What is a good estimate for the number of cars in town? Explain how you determined your answer.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**a** **S-CP.B.8**	Student is not able to calculate a probability.	Student does not set up as a multiplication rule calculation.	Student recognizes the calculation method but substitutes the numbers incorrectly or does not explain solution approach sufficiently.	Student recognizes the conditional probabilities and uses the multiplication rule to combine.
	b **S-CP.B.8**	Student is not able to calculate a probability.	Student constructs a probability tree or table but is not able to finish it or does not have the correct outcomes.	Student recognizes the calculation method but does not utilize probability of first serve not landing in or does not explain the solution approach sufficiently.	Student recognizes the conditional probabilities and uses the multiplication rule to combine.
	c **S-CP.B.8**	Student is not able to calculate a probability or assumes 0.50.	Student does not weight probabilities of winning points by probabilities of serves landing in.	Student does not realize (a) and (b) are mutually exclusive events. <u>AND/OR</u> Student attempts a new, incorrect approach.	Student sums answers from (a) and (b).
2	**S-CP.B.9**	Student is not able to calculate a probability.	Student uses incorrect probability rules, such as $\frac{1}{5}$.	Student fails to consider the 88 nonvenomous species.	Student uses combinations to set up the problem.

3	**a** S-CP.B.9	Student is not able to approach problem.	Student calculates a probability.	Student employs multiplication rule but does not allow for repeats or does not take into account number in each category.	Student employs multiplication rule.
	b S-CP.B.9	Student is not able to approach problem.	Student calculates a probability.	Student employs multiplication rule but allows for repeats.	Student employs multiplication rule and does not allow repeats (could begin second set at 10 again; could fail to include zero in first set).
	c S-CP.B.9	Student does not address the question (e.g., "yes").	Student specifies an answer consistent with the approach but is not able to explain why in terms of order of outcomes.	Student specifies permutations but is not able to explain why. AND/OR Student incorrectly identifies a permutation approach as using combinations.	Student specifics permutations and relates to the distinctness of the outcomes based on order.
4	**a** S-MD.A.1	Student does not produce a graph.	Student produces a graph of x values that does not consider $p(x)$ values.	Student produces a graph but does not label both axes.	Student produces a graph of $p(x)$ vs. x with appropriate labels on both axes.
	b S-MD.A.2	Student does not use provided information.	Student calculates an average of the x values and does not use $p(x)$ information.	Student does not show work or has a substantial calculation error.	Student calculates expected value correctly and shows work.
	c S-MD.A.2	Student provides an incorrect statement about expected value (e.g., most probable value).	Student merely restates a definition/calculation of expected value without clarifying the interpretation.	Student indicates an average but does not sufficiently put statements in context (e.g., no "long-run").	Student interprets expected value as a long-run average in context.
5	**a** S-MD.A.4	Student provides values that do not sum to one.	Student does not use the counts provided to create the relative frequencies (e.g., assumes 0.25 for each).	Student creates a table of relative frequencies, but calculation details are unclear.	Student creates a table of relative frequencies, and calculation details are clear.
	b S-MD.A.1	Student creates a variable but is not considering probability distributions.	Student does not convey the mapping of outcomes to numbers, giving more of a definition of random outcomes.	Student discusses the 0, 1, 2, 3 outcomes but does not define a random variable.	Student creates a mapping of outcomes to numerical values.

	c **S-MD.A.4**	Student does not calculate an expected value.	Student calculates an expected value but not using 3 in the last category (e.g., ignores category or uses a number larger than 3).	Student makes a calculation error or does not show sufficient calculation details.	Student correctly calculates an expected value using 3 as the last outcome and shows sufficient calculation details.
	d **S-MD.A.3**	Student does not relate comments to calculation details of expected value.	Student claims the expected value will be smaller because the probabilities of different outcomes will be smaller.	Student indicates the expected value will be larger but does not have a clear explanation.	Student explains reasoning that larger numbers increase the weighted average.
	e **S-MD.A.4**	Student provides a response that is not in "number of cars."	Student does not utilize the information of expected value or the given probability distribution (e.g., assumes two cars per household).	Student does not explain solution approach but finds correct answer.	Student multiplies the expected value by the number of households, and calculation details are clear.

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1. In the game of tennis, one player serves to a second player to start a point. If the server misses landing the first serve (the ball is hit out of bounds and cannot be played), the server is allowed a second serve. Suppose a particular tennis player, Chris, has a 0.62 probability of a first serve landing in bounds and a 0.80 probability of a second serve landing in bounds. Once a serve has landed in bounds, the players take turns hitting the ball back and forth until one player hits the ball out of bounds or into the net. The player who hits the ball out or into the net loses the point, and the other player wins the point. For Chris, when the first serve lands in bounds, he has a 75% chance of winning the point; however, he has only a 22% chance of winning the point on his second serve.

   a. Calculate the probability that Chris lands his first serve in bounds and then goes on to win the point for a randomly selected point.

   P(first serve in)P (wins point | first serve in)= 0.62 (0.75)=0.465

   b. Calculate the probability that Chris misses the first serve, lands the second serve, and then wins the point.

   P(first serve out and second serve in)P(wins point|second serve in)= 0.38(0.80)(0.22)
   =0.06688.

   c. Calculate the probability that Chris wins a randomly selected point.

   P(first serve in)P(wins point|first serve in)+P(second serve)P(wins point|second serve)
   = 0.465+0.06688= 0.53188.

2. In Australia, there are approximately 100 species of venomous snakes, but only about 12 have a deadly bite. Suppose a zoo randomly selects one snake from each of five different species to show visitors. What is the probability that exactly one of the five snakes shown is deadly?

$\frac{C(12,\ 1)C(88,4)}{C(100,5)} \approx 0.372.$

3. In California (CA), standard license plates are currently of the form: 1 number–3 letters–3 numbers. Assume the numbers are 0–9 and the letters are A–Z.

   a. In theory, how many different possible standard CA license plates are there, assuming we can repeat letters and numbers?

   $10 \times 26^3 \times 10^3 = 175{,}760{,}000$

   b. How many different possible standard CA license plates are there if we are not allowed to repeat any letters or numbers?

   $10 \times 26 \times 25 \times 24 \times 9 \times 8 \times 7 = 78{,}624{,}000$ (may start second sequence of numbers at 10 again)

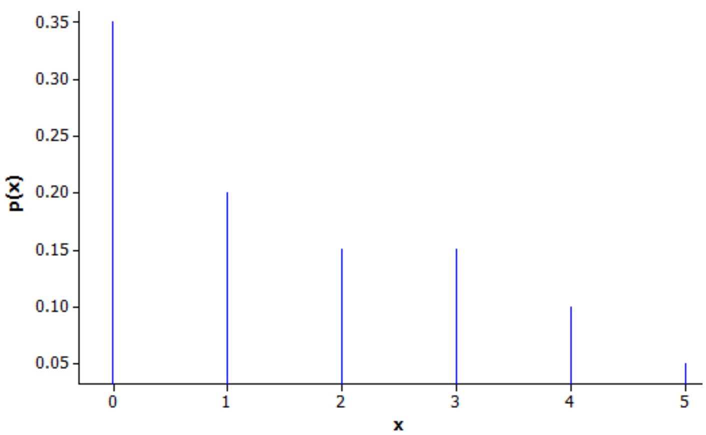
   c. For part (b), did you use permutations or combinations to carry out the calculation? Explain how you know.

   I used permutation because order matters.

4. When there is a problem with a computer program, many people call a technical support center for help. Several people may call at once, so the center needs to be able to have several telephone lines available at the same time. Suppose we want to consider a random variable that is the number of telephone lines in use by the technical support center of a software manufacturer at a particular time of day. Suppose that the probability distribution of this random variable is given by the following table:

x	0	1	2	3	4	5
$p(x)$	0.35	0.20	0.15	0.15	0.10	0.05

a. Produce a graph of the probability distribution for this random variable, including all relevant labels.



b. Calculate the expected value of the random variable.

Expected value = 0(0.35) + 1(0.20) + 2(0.15) + 3(0.15) + 4(0.10) + 5(0.05) = 1.6 telephone lines.

c. Explain how to interpret this expected value in context.

If we were to observe the number of phone lines in use over a very large number of days, the average number of lines in use will approach 1.6 in the long run.

5. The following table lists the number of U.S. households (in thousands) with 0 vehicles, 1 vehicle, 2 vehicles, or 3 or more vehicles for all households responding to the 2009 National Household Travel Survey.

No vehicle	One vehicle	Two vehicles	Three or more vehicles
9,828	36,509	41,077	25,668

a. Use these data to create a table of relative frequencies that could be used as estimates of the probability distribution of number of vehicles for a randomly selected U.S. household (that responded to the survey).

No vehicle	One vehicle	Two vehicles	Three or more vehicles	*Total*
0.0869	0.323	0.363	0.227	113,082 (thousand)

b. Suppose you want to examine the distribution of the number of vehicles in all U.S. households. Explain how you could represent this as a random variable.

We could let x represent the number of vehicles, so $x = 0, 1, 2, 3, \ldots$.

c. Assume for the moment that the last column corresponds to exactly three vehicles. Calculate the expected number of vehicles per household.

$0(0.0869) + (1)(0.323) + 2(0.363) + 3(0.227) = 1.73$ *vehicles.*

d. Now reconsider the last category. Suppose we were to find the information for the actual number of vehicles for these households. Would the expected number of vehicles per household with this new information be larger or smaller than the expected value you found in (c)? Explain your reasoning.

This would give us a larger expected value because we would decrease the probability of multiplying 3 vehicles and then add in larger numbers of vehicles.

e. Suppose a town has about 4,500 households. What is a good estimate for the number of cars in town? Explain how you determined your answer.

We expect 1.73 vehicles per household, so 4,500 households would correspond to $1.73 \times 4{,}500 = 7{,}785$ vehicles.