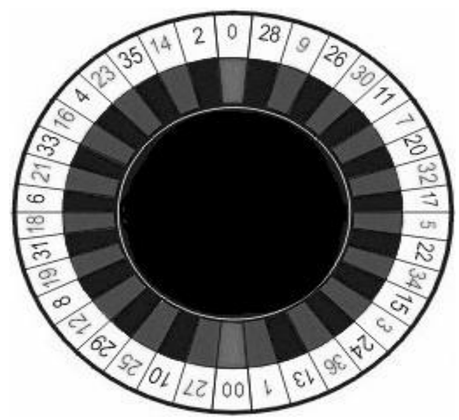
Name ____ Date ______________________

1. An assembler of computer routers and modems uses parts from three sources. Company A supplies 60% of the parts, company B supplies 30% of the parts, and company C supplies the remaining 10% of the parts. From past experience, the assembler knows that 3% of the parts supplied by company A are defective, 5% of the parts supplied by company B are defective, and 8% of the parts supplied by company C are defective. If a part is selected at random, what is the probability it is defective?

2. In the game of roulette, a wheel with different slots is spun. A ball is placed in the wheel and bounces around until it settles in one of the 38 slots, all of equal size. There are 18 red slots, 18 black slots, and 2 green slots. Suppose it costs $\$1.00$ to play one round of the game. A "color bet" allows you to pick either red or black; if the ball lands on your color, you win $\$2.00$. If the ball lands on either of the other two colors, you do not win any money.



   a. In this game, for each spin you win either $\$2.00$ or $\$0.00$. Determine the probabilities of winning each amount.

   b. Calculate the expected winnings in one spin of the wheel.

   c. Are your expected winnings in one spin of the wheel larger or smaller than the $\$1.00$ bet?

d. A casino's profit is equal to the amount of bets minus the amount of winnings. For each spin of the Roulette wheel, the expected profit is $\$1.00$ minus your expected winnings. What is the casino's expected profit? Explain why the game of roulette is still attractive to a professional casino even though the expected profit to the casino is such a small amount on each spin.

e. The slots are also numbered 0, 00, $1–36$. If you bet on a number and win, you win $\$36.00$. Which bet is better for you: a number bet or a color bet? Explain your decision.

f. Suppose you plan to construct a wheel with 12 slots: 5 red, 5 black, and 2 green. You plan to pay $\$5.00$ in winnings if someone picks a color (red or black) and the ball lands on that color. How much should you charge someone to play (the bet amount) so that you have created a fair game?

3. In New York, the Mega Ball jackpot lottery asks you to pick 5 numbers (integers) from 1 to 59 (the "upper section") and then pick a Mega Ball number from 1 to 35 (the "lower section"). You win $\$10{,}000$ if you match exactly 4 of the 5 numbers from the upper section and match the Mega Ball number from the lower section. The winning number(s) for each section are chosen at random without replacement. Determine the probability of correctly choosing exactly 4 of the winning numbers and the Mega Ball number.

4. A blood bank is screening a large population of donations for a particular virus. Suppose 5% of the blood donations contain the virus. Suppose you randomly select 10 bags of donated blood to screen. You decide to take a small amount from each of the bags and pool these all together into one sample. If that sample shows signs of the virus, you then test all of the original 10 bags individually. If the combined sample does not contain the virus, then you are done after just the one test.

   a. Determine the expected number of tests to screen 10 bags of donated blood using this strategy.

   b. Does this appear to be an effective strategy? Explain how you know.

5. Twenty-five sixth-grade students entered a math contest consisting of 20 questions. The student who answered the greatest number of questions correctly will receive a graphing calculator. The rules of the contest state that if two or more students tie for the greatest number of correct answers, one of these students will be chosen to receive the calculator.

   No student answered all 20 questions correctly, but four students (Allan, Beth, Carlos, and Denesha) each answered 19 questions correctly.

   What would be a fair way to use two coins (a dime and a nickel) to decide which student should get the calculator? Explain what makes your method fair.

6. A cell phone company offers cell phone insurance for $\$7.00$ a month. If your phone breaks and you submit a claim, you must first pay a $\$200.00$ deductible before the cell phone company pays anything. Suppose the replacement cost for a phone is $\$650.00$. This means if you break your phone and have insurance, you have to pay only $\$200.00$ toward the replacement cost. This plan has a limit of two replacements; if you break your phone more than twice in one year, you pay for the full replacement cost for the additional replacements.

   Suppose that within one year, there is a 48% chance that you do not break your phone, a 36% chance that you break it once, a 12% chance that you break it twice, a 3% chance that you break it three times, and a 1% chance that you break it four times.

   a. Calculate the expected one-year cost of this insurance plan based on the monthly cost and the expected repair costs.

   b. Determine your expected replacement costs if you do not purchase insurance.

   c. Does this insurance plan seem to be a good deal? Explain why or why not.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**S-CP.B.8**	Student does not recognize this as a probability calculation or combines the given numbers in a nonsensical way.	Student identifies components of the problem but is not able to combine the provided information in an appropriate way. <u>AND/OR</u> Student shows how the components are related to the law of total probability but ends up with a probability outside $(0,1)$. Student may also confuse the law of total probability with Bayes's theorem.	Student identifies the correct conditional and unconditional probabilities but makes a calculation error, such as weighting the companies equally $(0.03 + 0.05 + 0.08)/3$.	Student identifies the correct conditional and unconditional probabilities and correctly performs the probability calculation. Student supports answer with formulas and/or diagrams.
2	**a** **S-MD.A.3** **S-MD.B.5**	Student assumes the probabilities are both 0.50 because there are only two outcomes (win, lose).	Student finds the probabilities for each slot but does not combine into the outcomes of winning and losing.	Student finds the probabilities of winning and losing but does not relate to dollar amounts.	Student finds the probabilities for $\$2.00$ and $\$0.00$ based on the roulette slots being equally likely.
	b **S-MD.B.5**	Student is not able to determine an expected value.	Student attempts to find an expected value but does not use the probability distribution from (a).	Student uses the probability distribution from (a) to calculate an expected amount but then rounds to $\$1.00$.	Student uses the probability distribution from (a) to calculate an expected amount.

	c **S-MD.B.5**	Student is not able to compare the values.	Student does not use the expected winnings calculation.	Student makes a comparison but not the correct values. For example, student tries to utilize "net winnings" instead.	Student compares the answer in (b) to $\$1.00$ and indicates which is larger.
	d **S-MD.A.2**	Student only focuses on the casino using unfair games.	Student brings in outside information but does not argue based on expected profit for large numbers of customers.	Student understands that the casino can be confident they will win $\$0.05$ per spin, on average, but does not relate this to large numbers of spins.	Student understands that the answer to (c) is not large but argues based on the law of large numbers.
	e **S-MD.B.7**	Student does not consider the probability distribution and the change in the amount won.	Student compares the bets but does not calculate expected winnings for the numbered bets.	Student argues based on expected values but does not compare bets or does not show calculation details to support answer.	Student calculates the expected winnings for the numbered bet and makes a comparative statement.
	f **S-MD.B.6**	Student does not use the given probability distribution.	Student calculates the probability of winning but does not relate to the amount bet.	Student calculates the probability of winning but assumes a $\$1.00$ bet.	Student calculates the expected winnings and indicates that the cost should be at least that amount.
3	**S-CP.B.9**	Student brings in outside information without using the provided probability distribution.	Student attempts to use the multiplication rule but is not able to correctly calculate the probability.	Student determines the probability as one out of the number of possible combinations but does not separate the upper section and lower section.	Student determines the probability as one out of the number of possible combinations.
4	**a** **S-MD.B.7**	Student is not able to calculate a probability or an expected value from the given information.	Student only calculates the probability of needing a retest. Student is not able to convert to an expected number of tests.	Student considers the initial test and retest but does not calculate the probability of needing a retest correctly or does not have the correct outcomes for $x =$ number of tests (e.g., 1 and 10).	Student calculates the expected number of tests considering the initial test and the retests.
	b **S-MD.B.7**	Student does not answer based on expected number of tests between the two strategies.	Student considers the expected number of tests from (a) but does not compare to 10.	Student finds the strategy to be effective but does not clearly explain why.	Student compares the expected number of tests to 10.

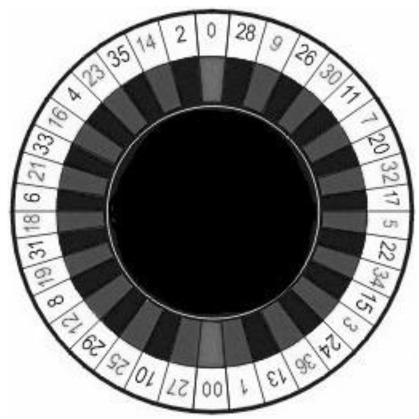
5	**S-MD.B.6**	Student is not able to describe a fair way to decide who should be awarded the calculator.	Student describes a method that is not fair or that does not use two coins.	Student describes a fair method using two coins, but the explanation does not indicate that each student is equally likely to be awarded the calculator.	Student describes a fair method using two coins, and the explanation indicates that each student is equally likely to be awarded the calculator.
6	**a** **S-MD.B.7**	Student does not calculate an expected cost.	Student considers the costs but does not combine them with the probabilities.	Student makes a calculation error (e.g., does not consider deductible) or does not show sufficient calculation detail.	Student combines the information to calculate an expected repair cost.
	b **S-MD.B.7**	Student is not able to calculate an expected cost.	Student states an expected cost but not based on the probabilities of number of phone breaks.	Student calculates an expected cost but makes a calculation error (e.g., considers information from the plan) or does not show sufficient calculation detail.	Student calculates the expected costs based on probabilities of breaking the phone 0–4 times.
	c **S-MD.B.7**	Student brings in outside information, and the argument is not based on previous calculations.	Student answers based on probability distributions rather than expected costs.	Student answers based on expected costs but does not clearly compare the answers to (a) and (b).	Student compares the two expected values.

Name ______________________________ Date ________________

1. An assembler of computer routers and modems uses parts from three sources. Company A supplies 60% of the parts, company B supplies 30% of the parts, and company C supplies the remaining 10% of the parts. From past experience, the assembler knows that 3% of the parts supplied by company A are defective, 5% of the parts supplied by company B are defective, and 8% of the parts supplied by company C are defective. If a part is selected at random, what is the probability it is defective?

   $P(\text{defective}) = P(\text{defective}|A)P(A) + P(\text{defective}|B)P(B) + P(\text{defective}|C)P(C)$
   $= 0.6(0.03) + 0.3(0.05) + 0.1(0.08) = 0.041.$

2. In the game of roulette, a wheel with different slots is spun. A ball is placed in the wheel and bounces around until it settles in one of the 38 slots, all of equal size. There are 18 red slots, 18 black slots, and 2 green slots. Suppose it costs $\$1.00$ to play one round of the game. A "color bet" allows you to pick either red or black; if the ball lands on your color, you win $\$2.00$. If the ball lands on either of the other two colors, you do not win any money.



   a. In this game, for each spin you win either $\$2.00$ or $\$0.00$. Determine the probabilities of winning each amount.

      $P(\$2) = \frac{18}{38} = 0.4737.$
      $P(\$0) = \frac{20}{38} = 0.5263.$

   b. Calculate the expected winnings in one spin of the wheel.

      expected winnings $= 2(18/38) + 0(20/38) = \frac{36}{38} = 0.9474.$

   c. Are your expected winnings in one spin of the wheel larger or smaller than the $\$1.00$ bet?

      This is smaller than the $1.00 bet (about 5 cents).

d. A casino's profit is equal to the amount of bets minus the amount of winnings. For each spin of the Roulette wheel, the expected profit is $1.00 minus your expected winnings. What is the casino's expected profit? Explain why the game of roulette is still attractive to a professional casino even though the expected profit to the casino is such a small amount on each spin.

The casino's expected winnings are about 5 cents per spin. The law of large numbers states that if they can get enough people to play, the average winnings will be very close to the expected winnings. So, the casino will be pretty sure of being close to the expected winnings. Even a small amount of expected winnings will add up over a large number of spins.

e. The slots are also numbered 0, 00, 1–36. If you bet on a number and win, you win $36.00. Which bet is better for you: a number bet or a color bet? Explain how you are deciding.

expected winnings $= 36\left(\frac{1}{38}\right) = \frac{36}{38}$.

The expected winnings are the same for the two bets. (So, you could argue that there isn't a difference, or you could argue in terms of the thrill vs. security of the bets.)

f. Suppose you plan to construct a wheel with 12 slots: 5 red, 5 black, and 2 green. You plan to pay $5.00 in winnings if someone picks a color (red or black) and the ball lands on that color. How much should you charge someone to play (the bet amount) so that you have created a fair game?

expected winnings $= 5\left(\frac{5}{12}\right) = \frac{25}{12}$. *So, you should charge approximately $2.08 to break even. You should also accept $2.09 to cover the rounding from $2.083.*

3. In New York, the Mega Ball jackpot lottery asks you to pick 5 numbers (integers) from 1 to 59 (the "upper section") and then pick a Mega Ball number from 1 to 35 (the "lower section"). You win $10,000 if you match exactly 4 of the 5 numbers from the upper section and match the Mega Ball number from the lower section. The winning number(s) for each section are chosen at random without replacement. Determine the probability of correctly choosing exactly 4 of the winning numbers and the Mega Ball number.

P(4 winning numbers and 1 Mega Ball) $= \left(\frac{1}{C(56,5)}\right) C(35,1)$.

The probability of correctly choosing exactly 4 of the winning numbers and the Mega Ball number is 0.00000000748 (7.48 x10⁻⁹)

4. A blood bank is screening a large population of donations for a particular virus. Suppose 5% of the blood donations contain the virus. Suppose you randomly select 10 bags of donated blood to screen. You decide to take a small amount from each of the bags and pool these all together into one sample. If that sample shows signs of the virus, you then test all of the original 10 bags individually. If the combined sample does not contain the virus, then you are done after just the one test.

   a. Determine the expected number of tests to screen 10 bags of donated blood using this strategy.

   If none of the blood donations have the virus, then you only have to conduct one test. The probability that none of the (independent) people have the virus is $(1-0.05)^{10} = 0.5987$.
   So, the expected number of tests is $1(0.5987) + 11(1-0.5987) = 5.01$.

   b. Does this appear to be an effective strategy? Explain how you know.

   This expected value in part (a) is much less than 10, so this seems to be an effective strategy. In the long run, you will perform fewer tests.

5. Twenty-five sixth-grade students entered a math contest consisting of 20 questions. The student who answered the greatest number of questions correctly will receive a graphing calculator. The rules of the contest state that if two or more students tie for the greatest number of correct answers, one of these students will be chosen to receive the calculator.

   No student answered all 20 questions correctly, but four students (Allan, Beth, Carlos, and Denesha) each answered 19 questions correctly.

   What would be a fair way to use two coins (a dime and a nickel) to decide which student should get the calculator? Explain what makes your method fair.

   Answers will vary. One possible response is to toss the coins, and if both coins land heads up (HH), Allan gets the calculator. If the dime lands heads up and the nickel lands tails up (HT), Beth gets the calculator. If the dime lands tails up and the nickel lands heads up (TH), Carlos gets the calculator. If both coins land tails up (TT), Denesha gets the calculator. This method is fair because each of the four students has the same chance (a probability of $\frac{1}{4}$) of getting the calculator.

6. A cell phone company offers cell phone insurance for $7.00 a month. If your phone breaks and you submit a claim, you must first pay a $200.00 deductible before the cell phone company pays anything. Suppose the replacement cost for a phone is $650.00. This means if you break your phone and have insurance, you have to pay only $200.00 toward the replacement cost. This plan has a limit of two replacements; if you break your phone more than twice in one year, you pay for the full replacement cost for the additional replacements.

   Suppose that within one year, there is a 48% chance that you do not break your phone, a 36% chance that you break it once, a 12% chance that you break it twice, a 3% chance that you break it three times, and a 1% chance that you break it four times.

   a. Calculate the expected one-year cost of this insurance plan based on the monthly cost and the expected repair costs.

      If you do not break your phone, then you pay $7 × 12 = $84.
      If you break your phone once, then you pay $200 + $84 = $284.
      If you break your phone twice, then you pay $400 + $84 = $484.
      If you break your phone three times, then you pay $484 + $650 = $1134.
      If you break your phone four times, then you pay $1134 + $650 = $1784.
      The probability distribution of costs is as follows:

$84	$284	$484	$1134	$1784
0.48	0.36	0.12	0.03	0.01

      Therefore, your expected costs are
      0.48(84) + 0.36(284) + 0.12(484) + 0.03(1134) + 0.01(1784) = $252.50.

   b. Determine your expected replacement costs if you do not purchase insurance.

      0.48(0) + 0.36(650) + 0.12(1300) + 0.03(1950) + 0.01(2600) = $474.50.

   c. Does this insurance plan seem to be a good deal? Explain why or why not.

      Yes, the long-run average costs are considerably less with the insurance plan than without it.