Lesson 26: Projecting a 3-D Object onto a 2-D Plane

Exercises

1. Describe the set of points $(8t, 3t)$, where t represents a real number.

2. Project the point $(8, 3)$ onto the line $x = 1$.

3. Project the point $(8, 3)$ onto the line $x = 5$.

4. Project the point $(-1, 4,\ 5)$ onto the plane $y = 1$.

5. Project the point $(9, 5, -8)$ onto the plane $z = 3$.

Problem Set

1. A cube in 3-D space has vertices $\begin{pmatrix}10\\10\\10\end{pmatrix}, \begin{pmatrix}13\\10\\10\end{pmatrix}, \begin{pmatrix}10\\13\\10\end{pmatrix}, \begin{pmatrix}10\\10\\13\end{pmatrix}, \begin{pmatrix}13\\13\\10\end{pmatrix}, \begin{pmatrix}13\\10\\13\end{pmatrix}, \begin{pmatrix}10\\13\\13\end{pmatrix}, \begin{pmatrix}13\\13\\13\end{pmatrix}$.
   a. How do we know that these vertices trace a cube?
   b. What is the volume of the cube?
   c. Let $z = 1$. Find the eight points on the screen that represent the vertices of this cube (some may be obscured).
   d. What do you notice about your result in part (c)?

2. An object in 3-D space has vertices $\begin{pmatrix}1\\5\\0\end{pmatrix}, \begin{pmatrix}0\\6\\0\end{pmatrix}, \begin{pmatrix}0\\5\\1\end{pmatrix}, \begin{pmatrix}-1\\5\\0\end{pmatrix}, \begin{pmatrix}0\\4\\0\end{pmatrix}$.
   a. What kind of shape is formed by these vertices?
   b. Let $y = 1$. Find the five points on the screen that represent the vertices of this shape.

3. Consider the shape formed by the vertices given in Problem 2.
   a. Write a transformation matrix that will rotate each point around the y-axis θ degrees.
   b. Project each rotated point onto the plane $y = 1$ if $\theta = 45°$.
   c. Is this the same as rotating the values you obtained in Problem 3 by $45°$?

4. In technical drawings, it is frequently important to preserve the scale of the objects being represented. In order to accomplish this, instead of a perspective projection, an orthographic projection is used. The idea behind the orthographic projection is that the points are translated at right angles to the screen (the word stem *ortho-* means *straight* or *right*). To project onto the xy-plane for instance, we can use the matrix $\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\end{pmatrix}$.
   a. Project the cube in Problem 1 onto the xy-plane by finding the 8 points that correspond to the vertices.
   b. What do you notice about the vertices of the cube after projecting?
   c. What shape is visible on the screen?
   d. Is the area of the shape that is visible on the screen what you expected from the original cube? Explain.
   e. Summarize your findings from parts (a)–(d).
   f. State the orthographic projection matrices for the xz-plane and the yz-plane.
   g. In regard to the dimensions of the orthographic projection matrices, what causes the outputs to be two-dimensional?

5. Consider the point $A = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$ in the field of view from the origin through the plane $z = 1$.
   a. Find the projection of A onto the plane $z = 1$.
   b. Find a 3×3 matrix P such that PA finds the projection of A onto the plane $z = 1$.
   c. How does the matrix change if instead of projecting onto $z = 1$, we project onto $z = c$, for some real number $c \neq 0$?
   d. Find the scalars that will generate the image of A onto the planes $x = c$ and $y = c$, assuming the image exists. Describe the scalars in words.

Extension:

6. Instead of considering the rotation of a point about an axis, consider the rotation of the camera. Rotations of the camera will cause the screen to rotate along with it, so that to the viewer, the screen appears immobile.
   a. If the camera rotates θ_x around the x-axis, how does the computer world appear to move?
   b. State the rotation matrix we could use on a point A to simulate rotating the camera and computer screen by θ_x about the x-axis but in fact keeping the camera and screen fixed.
   c. If the camera rotates θ_y around the y-axis, how does the computer world appear to move?
   d. State the rotation matrix we could use on a point A to simulate rotating the camera and computer screen by θ_y about the y-axis but in fact keeping the camera and screen fixed.
   e. If the camera rotates θ_z around the z-axis, how does the computer world appear to move?
   f. State the rotation matrix we could use on a point A to simulate rotating the camera and computer screen by θ_z about the z-axis but in fact keeping the camera and screen fixed.
   g. What matrix multiplication could represent the camera starting at a relative angle $(\theta_x, \theta_y, \theta_z)$? Apply the transformations in the order z-y-x. Do not find the product.