Lesson 22: Linear Transformations of Lines

Classwork

Opening Exercise

* 1. Find parametric equations of the line through point in the direction of vector .
  2. Find parametric equations of the line through point in the direction of vector .

Exercises 1–3

1. Consider points and , and define a linear transformation by Find parametric equations to describe the image of line under the transformation .
2. The process that we developed for images of lines in also applies to lines in . Consider points and Define a linear transformation by Find parametric equations to describe the image of line under the transformation .

1. Not only is the image of a line under a linear transformation another line, but the image of a line segment under a linear transformation is another line segment. Let and be as specified in Exercise 2. Find parametric equations to describe the image of segment under the transformation.

Lesson Summary

We can find vector and parametric equations of a line in the plane or in space if we know two points that the line passes through, and we can find parametric equations of a line segment in the plane or in space by restricting the values of in the parametric equations for the line.

* Let be a line in the plane that contains point and . Then a direction vector is given by , and an equation in vector form that represents line is

for all real numbers.

Parametric equations that represent line are

Parametric equations that represent segment are

* Let be a line in space that contains points and Then a direction vector is given by , and an equation in vector form that represents line is

for all real numbers .

Parametric equations that represent line are

Parametric equations that represent segment are

* The image of a line in the plane under a linear transformation is given by

, for all real numbers .

* The image of a line in space under a linear transformation is given by

for all real numbers

Problem Set

1. Find parametric equations of the line through points and in the plane.
   1. ,
   2. ,
   3. ,
2. Find parametric equations of the line through points and in space.
   1. ,
   2. ,
3. Find parametric equations of segment through points and in the plane.
   1. ,
   2. ,
   3. ,
4. Find parametric equations of segment through points and in space.
   1. ,
   2. ,
   3. ,
5. Jeanine claims that the parametric equations and describe the line through points and . Is she correct? Explain how you know.
6. Kelvin claims that the parametric equations and describe the line through points and . Is he correct? Explain how you know.
7. LeRoy claims that the parametric equations and describe the line through points and . Is he correct? Explain how you know.
8. Miranda claims that the parametric equations and describe the line through points and . Is she correct? Explain how you know.
9. Find parametric equations of the image of the line under the transformation for the given points , and matrix
   1. , ,
   2. , ,
   3. , ,
10. Find parametric equations of the image of the line under the transformation for the given points , , and matrix
    1. , ,
    2. , ,
    3. , ,
11. Find parametric equations of the image of the segment under the transformation for the given points , , and matrix
    1. , ,
    2. , ,
    3. , ,
12. Find parametric equations of the image of the segment under the transformation for the given points , and matrix
    1. , ,
    2. , ,
    3. , ,