



Topic D:

Vectors in Plane and Space

N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5, N-VM.C.11

Focus Standards:	N-VM.A.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g. \mathbf{v} , $ \mathbf{v} $, $\ \mathbf{v}\ $, v).
	N-VM.A.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
	N-VM.A.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
	N-VM.B.4	(+) Add and subtract vectors. <ol style="list-style-type: none"> Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that magnitude of a sum of two vectors is typically not the sum of the magnitudes. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w}, with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
	N-VM.B.5	(+) Multiply a vector by a scalar. <ol style="list-style-type: none"> Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\ = c v$. Compute the direction of $c\mathbf{v}$ knowing that when $c v \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} for $(c > 0)$ or against \mathbf{v} for $(c < 0)$.
	N-VM.C.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Instructional Days: 8

- Lesson 17:** Vectors in the Coordinate Plane (P)¹
- Lesson 18:** Vectors and Translation Maps (P)
- Lesson 19:** Directed Line Segments and Vectors (P)
- Lesson 20:** Vectors and Stone Bridges (E)
- Lesson 21:** Vectors and the Equation of a Line (S)
- Lesson 22:** Linear Transformations of Lines (S)
- Lessons 23–24:** Why Are Vectors Useful? (P, S)

Topic D opens with a formal definition of a vector (the motivation and context for it is well in place at this point), and the arithmetical work for vector addition, subtraction, scalar multiplication, and vector magnitude is explored along with the geometrical frameworks for these operations (**N-VM.A.1**, **N-VM.A.2**, **N-VM.B.4**, **N-VM.B.5**).

Lesson 17 introduces vectors in terms of translations. Students use their knowledge of transformations to represent vectors as arrows with an initial point and a terminal point. They calculate the magnitude of a vector, add and subtract vectors, and multiply a vector by a scalar. Students interpret these operations geometrically and compute them componentwise (**N-VM.A.1**, **N-VM.A.3**, **N-VM.B.4**, **N-VM.B.5**). Lesson 18 builds on vectors as shifts by relating them to translation maps studied in prior lessons. The connection between matrices and vectors becomes apparent in this lesson as a notation for vectors is introduced that recalls matrix notation. This lesson's focus extends vector addition, subtraction, and scalar multiplication to \mathbb{R}^3 .

Lesson 19 introduces students to directed line segments and how to subtract initial point coordinates from terminal point coordinates to find the components of a vector. Vector arithmetic operations are reviewed, and the parallelogram rule is introduced. Students study the magnitude and direction of vectors. In Lesson 20, students apply vectors to real-world applications as they look at vector effects on ancient stone arches and try to create their own.

In Lesson 21, students describe a line in the plane using vectors and parameters, and then they apply this description to lines in \mathbb{R}^3 . The shift to describing a line using vectors to indicate the direction of the line requires that students think geometrically about lines in the plane instead of algebraically. Lesson 21 poses the question, “Is the image of a line under a linear transformation a line?” Students must extend the process of finding parametric equations for a line in \mathbb{R}^2 and \mathbb{R}^3 (**N-VM.C.11**). Lessons 21 and 22 help students see the coherence between the work that they have done with functions and how that relates to vectors written using parametric equations. This sets the mathematical foundation that students will need to understand the definition of vectors. Vectors are generally described as a quantity that has both a magnitude and a direction. In Lessons 21 and 22, students perform linear transformations in two and three dimensions by writing parametric equations. The most basic definition of a vector is that it is a description of a shift or translation. Students will see that any physical operation that induces a shift of some kind is often thought of as a vector. Hence, vectors are prevalent in mathematics, science, and engineering. For example, force is often interpreted as a vector in physics because a force exerted on an object is a push of some magnitude that causes the object to shift in some direction.

In Lessons 23 and 24, students will solve problems involving velocity as well as other quantities, such as force, that can be represented by vectors (**N-VM.A.3**). Students work with non-right triangles and will continue to

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

work on adding and subtracting vectors (**N-VM.B.4**) but will interpret the resulting magnitude and direction within a context, for example with particle motion. Students discover the notion (using pictures only) that the method of transforming systems of linear equations while preserving the solution can be rephrased in terms of a series of linear transformations and translations.

In Topic D, students are making sense of vectors and using them to represent real world situations such as earthquakes, velocity, and force (MP.4). Students focus on vector arithmetic, determining the magnitude and direction of vectors and vector sums and products (MP.6), and constructing arguments about why the operations work the way that they do (MP.3).