Lesson 16: Solving General Systems of Linear Equations

Student Outcomes

- Students represent complicated systems of equations, including 4×4 and 5×5 systems of equations, using matrix equations in the form $Ax = b$, where A represents the coefficient matrix, x is the solution to the system, and b represents the constant matrix.
- Students use technology to calculate the inverse of matrices and use inverse matrix operations to solve complex systems of equations.

Lesson Notes

In this lesson, students apply their understanding from Lesson 14 and Lesson 15 to systems that are higher than 3-by-3 and systems in two- and three-dimensional space that are more complicated than those presented in previous lessons. They discover that, while it is difficult to geometrically describe linear transformations in four- or higher-dimensional space, the mathematics behind representing systems of equations as linear transformations using matrices is valid for higher degree space. They apply this reasoning to represent complicated systems of equations using matrices and use technology to solve the systems.

Classwork

Example 1 (15 minutes)

In this example, students see that a cubic function can be used to model scientific data comparing side length to volume (MP.4). Students should complete part (a) with a partner. After a few minutes, a selected pair should display its solution, demonstrating how to use substitution of the data points into the cubic equation to find the equations for the system. After verifying the correct system, the students should complete parts (b)–(d) with a partner. Technology that enables students to calculate the inverse of a matrix will be needed to complete part (c). Each student should write and solve the matrix equation independently and verify the answer with a partner, but students can work in small groups or pairs, especially if access to technology is limited. Part (e) should be completed as a teacher-led discussion. Students should be encouraged to critically assess the usefulness of the model. If time permits, students could create a two-column chart for display that lists positives and negatives for the model.

- In the problem, what do the ordered pairs represent?
  - *For each ordered pair, the first number is the greatest linear measurement of the irregularly-shaped object, and the second number represents the volume in cubic centimeters measured by water displacement.*
- So, how could we use the information from the problem to write a system of equations?
  - *Substitute each ordered pair into the $v(x)$ equation, and simplify the resulting equations. Specifically, substitute the first coordinate for x and the second coordinate for $v(x)$.*
- How would this look for the first point?
  - $3 = a(1^3) + b(1^2) + c(1) + d.$

- Once we have performed substitution with all the ordered pairs, what are we left with?
  - *Four equations with four unknowns.*
- And how can we represent our system of equations as a matrix equation?
  - *Use the equation* $Ax = b$*, where* A *represents the coefficient matrix for the system when all the equations are written in standard form,* $x = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ *and* b *represents the* $v(x)$ *values for the equations.*
- How can we use the matrix equation to write a model for the scientist's data?
  - *Isolate* x *by applying* A^{-1} *to both sides of the matrix equation.*
- Describe the measurements of the first object measured.
  - *Its greatest linear measurement is* 1 cm*, and its volume is* 3 *cubic centimeters.*
- Why did the scientist select a cubic equation to model the data?
  - *He generalized that there is usually a cubic relationship between the linear dimensions of an object and its volume.*
- How else could we determine whether a cubic equation would serve as a good model for the data?
  - *Answers will vary. Examples of appropriate responses would be that the points could be plotted to see if they seem to fit the pattern of a cubic function or that the data could be entered into a software program and a cubic regression performed.*
- How can you find the inverse of matrix A?
  - *The students should mention the software or application that has been used in previous lessons to calculate the inverse of 3-by-3 matrices.*
- Does A^{-1} have an inverse? If so, what is it?
  - *Yes, it is* $\begin{bmatrix} \frac{-1}{15} & \frac{1}{8} & \frac{-1}{12} & \frac{1}{40} \\ \frac{4}{5} & \frac{-11}{8} & \frac{3}{4} & \frac{7}{40} \\ \frac{-44}{15} & \frac{17}{4} & \frac{-5}{3} & \frac{7}{20} \\ 5 & -3 & 1 & \frac{-1}{5} \end{bmatrix}$.
- And how do we use the inverse matrix to solve the system?
  - $x = A^{-1}b = \begin{bmatrix} 0.175 \\ -1.225 \\ 4.45 \\ -0.4 \end{bmatrix}$

MP.4

- How do we use the solution to the system to write a cubic equation to model the data?
  - *Substitute the solution for* $a, b, c,$ *and* d *in the equation. The cubic equation that models the data is* $v(x) = 0.175x^3 - 1.225x^2 + 4.45x - 0.4$.
- How well does this model represent the scientist's data?
  - *Well because all the points fit the model.*
- Does this mean that a cubic model is the best model for this data?
  - *Not necessarily. It is possible that other models might be able to fit the data also.*

- What are some limitations of the model?
  - *Answers will vary but might include that there are only four data points and no indication that they are based on repeated measurements.*
- What could we recommend to the scientist if he wanted to strengthen his argument that a cubic model should represent the relationship between the greatest linear measure and the volume of the irregular objects?
  - *Answers will vary but might include having him take additional measurements and assess the fit of several types of models to the data set to see which model is the best fit.*

Example 1

A scientist measured the greatest linear dimension of several irregular metal objects. He then used water displacement to calculate the volume of each of the objects. The data he collected are $(1,3)$, $(2,5)$, $(4,9)$, and $(6,20)$, where the first coordinate represents the linear measurement of the object in centimeters, and the second coordinate represents the volume in cubic centimeters. Knowing that volume measures generally vary directly with the cubed value of linear measurements, he wants to try to fit this data to a curve in the form of $v(x) = ax^3 + bx^2 + cx + d$.

a. **Represent the data using a system of equations.**

$$3 = a + b + c + d$$
$$5 = 8a + 4b + 2c + d$$
$$9 = 64a + 16b + 4c + d$$
$$20 = 216a + 36b + 6c + d$$

b. **Represent the system using a matrix equation in the form $Ax = b$.**

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 64 & 16 & 4 & 1 \\ 216 & 36 & 6 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 9 \\ 20 \end{bmatrix}$$

c. **Use technology to solve the system.**

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 64 & 16 & 4 & 1 \\ 216 & 36 & 6 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 5 \\ 9 \\ 20 \end{bmatrix} = \begin{bmatrix} 0.175 \\ -1.225 \\ 4.45 \\ -0.4 \end{bmatrix}$$

d. **Based on your solution to the system, what cubic equation models the data?**

$$v(x) = 0.175x^3 - 1.225x^2 + 4.45x - 0.4$$

e. **What are some of the limitations of the model?**

It is based off of only four points, so the equation goes exactly through all the points.

Scaffolding:

- Advanced students can select an additional function type and use matrices to try to fit the data to their selected model.
- Provide students with a written copy of a worked example using a 3-by-3 system, and suggest they follow the same procedure to write and solve the matrix equation for this problem.
- Have students compute the cubic regression equation using software to verify that it matches the equation found using matrices.

Exercises 1-3 (20 minutes)

Allow students to complete the work in pairs. They should each complete the work and then compare answers. Some pairs may need more help or additional instruction. After a few minutes, discuss Exercise 1 part (a) to ensure that students are able to represent the situation using a matrix. After about 5 minutes, Exercise 1 could be reviewed and discussed to help students who are struggling with understanding how to apply the matrix to find the solutions in parts (b) and (c). After the discussion, students could complete Exercise 2 in pairs or small groups. It might be necessary to model for students how to compute matrices to powers (e.g., how to use software to find A^{30}).

Exercises 1–3

1. **An attendance officer in a small school district noticed a trend among the four elementary schools in the district. This district used an open enrollment policy, which means any student within the district could enroll at any school in the district. Each year, 10% of the students from Adams Elementary enrolled at Davis Elementary, and 10% of the students from Davis enrolled at Adams. In addition, 10% of the students from Brown Elementary enrolled at Carson Elementary, and 20% of the students from Brown enrolled at Davis. At Carson Elementary, about 10% of students enrolled at Brown, and 10% enrolled at Davis, while at Davis, 10% enrolled at Brown, and 20% enrolled at Carson. The officer noted that this year, the enrollment was 490, 250, 300, and 370 at Adams, Brown, Carson, and Davis, respectively.**

   a. **Represent the relationship that reflects the annual movement of students among the elementary schools using a matrix.**

$$A = \begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.7 & 0.1 & 0.2 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}$$

   b. **Write an expression that could be used to calculate the attendance one year prior to the year cited by the attendance officer. Find the enrollment for that year.**

   Expression $= A^{-1}b$*, where*

$$b = \begin{bmatrix} 490 \\ 250 \\ 300 \\ 370 \end{bmatrix}$$

$$A^{-1}b = \begin{bmatrix} 500 \\ 200 \\ 300 \\ 400 \end{bmatrix}$$

   Enrollment one year prior to cited data: 500 at Adams, 250 at Brown, 300 at Carson, and 400 at Davis.

   c. **Assuming that the trend in attendance continues, write an expression that could be used to calculate the enrollment two years after the year cited by the attendance officer. Find the attendance for that year.**

   Expression $= A^2b$

$$A^2b = \begin{bmatrix} 465.8 \\ 296.7 \\ 305.1 \\ 349.7 \end{bmatrix}$$

   d. **Interpret the results to part (c) in context.**

   The approximate enrollment at the schools would be 466, 297, 305, and 350 for Adams, Brown, Carson, and Davis, respectively.

Scaffolding:

- Define A as applying the enrollment trend from one year to the next year. Then have students determine an expression to represent applying the enrollment trend for 2 years, 3 years, and n years. Work with students to define A^{-1} as representing applying the enrollment trend one year backwards.
- Work step-by-step through part (a) with struggling students. Then encourage them to use similar reasoning to what was applied in Exercise 1 parts (b) and (c) to find the values in Exercise 2 parts (b) and (c).

2. Mrs. Kenrick is teaching her class about different types of polynomials. They have just studied quartics, and she has offered 5 bonus points to anyone in the class who can determine the quartic that she has displayed on the board. The quartic has 5 points identified: $(-6, 25)$, $(-3, 1)$, $\left(-2, \frac{7}{3}\right)$, $(0, -5)$, and $(3, 169)$. Logan really needs those bonus points and remembers that the general form for a quartic is $y = ax^4 + bx^3 + cx^2 + dx + e$. Can you help Logan determine the equation of the quartic?

a. Write the system of equations that would represent this quartic.

$$25 = 1296a - 216b + 36c - 6d + e$$
$$1 = 81a - 27b + 9c - 3d + e$$
$$\frac{7}{3} = 16a - 8b + 4c - 2d + e$$
$$-5 = e$$
$$169 = 81a + 27b + 9c + 3d + e$$

b. Write a matrix that would represent the coefficients of this quartic.

$$A = \begin{bmatrix} 1296 & -216 & 36 & -6 & 1 \\ 81 & -27 & 9 & -3 & 1 \\ 16 & -8 & 4 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 81 & 27 & 9 & 3 & 1 \end{bmatrix}$$

c. Write an expression that could be used to calculate coefficients of the equation.

Expression $= A^{-1}b$, *where*

$$b = \begin{bmatrix} 25 \\ 1 \\ \frac{7}{3} \\ -5 \\ 169 \end{bmatrix}$$

$$A^{-1}b = \begin{bmatrix} \frac{1}{3} \\ 3 \\ 7 \\ 1 \\ -5 \end{bmatrix}$$

d. Explain the answer in the context of this problem.

$$a = \frac{1}{3}, b = 3, c = 7, d = 1, c = -5$$

These are the coefficients of the quartic. The equation of the quartic is

$$y = \frac{1}{3}x^4 + 3x^3 + 7x^2 + x - 5$$

3. **The Fibonacci numbers are the numbers $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$. Each number beyond the second is the sum of the previous two.**

   Let $u_1 = \begin{bmatrix}1\\1\end{bmatrix}$, $u_2 = \begin{bmatrix}1\\2\end{bmatrix}$, $u_3 = \begin{bmatrix}2\\3\end{bmatrix}$, $u_4 = \begin{bmatrix}3\\5\end{bmatrix}$, $u_5 = \begin{bmatrix}5\\8\end{bmatrix}$, and so on.

   a. **Show that $u_{n+1} = \begin{bmatrix}0 & 1\\1 & 1\end{bmatrix} u_n$.**

   If we define the terms in the Fibonacci sequence as f_n, where $n = 1, 2, 3, 4, 5, \ldots$, then $u_1 = \begin{bmatrix}f_1\\f_2\end{bmatrix}$, $u_2 = \begin{bmatrix}f_2\\f_3\end{bmatrix}$, $u_3 = \begin{bmatrix}f_3\\f_4\end{bmatrix}$, $u_n = \begin{bmatrix}f_n\\f_{n+1}\end{bmatrix}$, and $u_{n+1} = \begin{bmatrix}f_{n+1}\\f_{n+2}\end{bmatrix}$. By definition, $f_{n+2} = f_{n+1} + f_n$, so $u_{n+1} = \begin{bmatrix}f_{n+1}\\f_{n+1} + f_n\end{bmatrix}$. Now, $\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix} u_n = \begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}f_n\\f_{n+1}\end{bmatrix} = \begin{bmatrix}f_{n+1}\\f_{n+1} + f_n\end{bmatrix}$, which is equivalent to u_{n+1}.

   b. **How could you use matrices to find u_{30}? Use technology to find u_{30}.**

   $$u_{30} = \begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}^{29}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}832040\\1346269\end{bmatrix}$$

   c. **If $u_n = \begin{bmatrix}165580141\\267914296\end{bmatrix}$, find u_{n-1}. Show your work.**

   $$u_{n-1} = \begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}^{-1} u_n = \begin{bmatrix}63245986\\102334155\end{bmatrix}$$

Closing (5 minutes)

Review Exercise 3 as a teacher-led discussion. Students should be encouraged to respond in writing to the questions provided:

- In what ways can matrix operations be useful in modeling real-world situations?
  - *Answers will vary but might include that they can be used to create models for data or to project trends forward and backward in time.*
- In what ways can matrix operations be useful in representing mathematical relationships like the numbers in the Fibonacci sequence?
  - *Answers will vary but might include that matrix operations can be used to determine values that would be very cumbersome to calculate by hand.*

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 16: Solving General Systems of Linear Equations

Exit Ticket

1. Anabelle, Bryan, and Carl are playing a game using sticks of gum. For each round of the game, Anabelle gives half of her sticks of gum to Bryan and one-fourth to Carl. Bryan gives one-third of his sticks to Anabelle and keeps the rest. Carl gives 40 percent of his sticks of gum to Anabelle and 10 percent to Bryan. Sticks of gum can be cut into fractions when necessary.

   a. After one round of the game, the players count their sticks of gum. Anabelle has 525 sticks of gum, Bryan has 600, and Carl has 450. How many sticks of gum will each player have after 2 more rounds of the game? Use a matrix equation to represent the situation, and explain your answer in context.

   b. How many sticks of gum did each player have at the start of the game? Use a matrix equation to represent the situation, and explain your answer in context.

Exit Ticket Sample Solutions

1. Anabelle, Bryan, and Carl are playing a game using sticks of gum. For each round of the game, Anabelle gives half of her sticks of gum to Bryan and one-fourth to Carl. Bryan gives one-third of his sticks to Anabelle and keeps the rest. Carl gives 40 percent of his sticks of gum to Anabelle and 10 percent to Bryan. Sticks of gum can be cut into fractions when necessary.

   a. After one round of the game, the players count their sticks of gum. Anabelle has 525 sticks of gum, Bryan has 600, and Carl has 450. How many sticks of gum will each player have after 2 more rounds of the game? Use a matrix equation to represent the situation, and explain your answer in context.

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{5} & \frac{1}{10} & \frac{1}{2} \end{bmatrix}^2 \begin{bmatrix} 525 \\ 600 \\ 450 \end{bmatrix} = x$$

$$x = \begin{bmatrix} 547\frac{3}{16} \\ 564\frac{7}{12} \\ 522\frac{1}{2} \end{bmatrix}$$

   Anabelle would have $547\frac{3}{16}$ sticks of gum, Bryan would have $564\frac{7}{12}$ sticks of gum, and Carl would have $522\frac{1}{2}$ sticks of gum.

   b. How many sticks of gum did each player have at the start of the game? Use a matrix equation to represent the situation, and explain your answer in context.

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{2}{5} & \frac{1}{10} & \frac{1}{2} \end{bmatrix}^{-1} \begin{bmatrix} 525 \\ 600 \\ 450 \end{bmatrix} = x$$

$$x = \begin{bmatrix} 600 \\ 600 \\ 300 \end{bmatrix}$$

   At the start of the game, Anabelle and Bryan each had 600 sticks of gum, and Carl had 300 sticks of gum.

Problem Set Sample Solutions

1. The system of equations is given:

$$1.2x + 3y - 5z + 4.2w + v = 0$$
$$6x = 5y + 2w$$
$$3y + 4.5z - 6w + 2v = 10$$
$$9x - y + z + 2v = -3$$
$$-4x + 2y - w + 3v = 1$$

   a. Represent this system using a matrix equation.

$$\begin{bmatrix} 1.2 & 3 & -5 & 4.2 & 1 \\ 6 & -5 & 0 & -2 & 0 \\ 0 & 3 & 4.5 & -6 & 2 \\ 9 & -1 & 1 & 0 & 2 \\ -4 & 2 & 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ -3 \\ 1 \end{bmatrix}$$

   b. Use technology to solve the system. Show your solution process, and round your entries to the tenths place.

$$\begin{bmatrix} x \\ y \\ z \\ w \\ v \end{bmatrix} = \begin{bmatrix} 1.2 & 3 & -5 & 4.2 & 1 \\ 6 & -5 & 0 & -2 & 0 \\ 0 & 3 & 4.5 & -6 & 2 \\ 9 & -1 & 1 & 0 & 2 \\ -4 & 2 & 0 & -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 10 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 1.3 \\ -1.5 \\ -2.5 \\ -1.1 \end{bmatrix}$$

2. A caterer was preparing a fruit salad for a party. She decided to use strawberries, blackberries, grapes, bananas, and kiwi. The total weight of the fruit was 10 pounds. Based on guidelines from a recipe, the weight of the grapes was equal to the sum of the weight of the strawberries and blackberries; the total weight of the blackberries and kiwi was 2 pounds; half the total weight of fruit consisted of kiwi, strawberries, and bananas; and the weight of the grapes was twice the weight of the blackberries.

   a. Write a system of equations to represent the constraints placed on the caterer when she made the fruit salad. Be sure to define your variables.

S = pounds of strawberries

B = pounds of blackberries

G = pounds of grapes

K = pounds of kiwi

Ba = pounds of bananas

$$S + B + G + K + Ba = 10$$
$$G = S + B$$
$$B + K = 2$$
$$K + S + Ba = 5$$
$$G = 2B$$

   b. Represent the system using a matrix equation.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & -2 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} S \\ B \\ G \\ K \\ Ba \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 2 \\ 5 \\ 0 \end{bmatrix}$$

c. Solve the system using the matrix equation. Explain your solution in context.

$$\begin{bmatrix} S \\ B \\ G \\ K \\ Ba \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & -2 & 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 0 \\ 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ \frac{5}{3} \\ \frac{10}{3} \\ \frac{1}{3} \\ 3 \end{bmatrix}$$

The fruit salad consisted of $\frac{5}{3}$ pounds of strawberries, $\frac{5}{3}$ pounds of blackberries, $\frac{10}{3}$ pounds of grapes, $\frac{1}{3}$ pound of kiwi, and 3 pounds of bananas.

d. How helpful would the solution to this problem likely be to the caterer as she prepares to buy the fruit?

It is useful as a general guideline, but the caterer is unlikely to buy the fruit in exactly the amount indicated by the problem. For instance, it is unlikely that she could purchase exactly $\frac{1}{3}$ pound of kiwi because it generally has to be purchased per fruit, not per ounce.

3. Consider the sequence $1, 1, 1, 3, 5, 9, 17, 31, 57, \ldots$ where each number beyond the third is the sum of the previous three. Let w_n be the points with the n^{th}, $(n+1)^{\text{th}}$, and $(n+2)^{\text{th}}$ terms of the sequence.

a. Find a 3×3 matrix A so that $Aw_n = w_{n+1}$ for each n.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

b. What is the 30^{th} term of the sequence?

$$w_{30} = A^{29} w_1$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{29} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 20603361 \\ 37895489 \\ 697006711 \end{bmatrix}$$

c. What is A^{-1}? Explain what A^{-1} represents in terms of the sequence. In other words, how can you find w_{n-1} if you know w_n?

$$A^{-1} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The first entry of w_{n-1} is the third entry from w_n minus the sum of the first two entries of w_n, the second entry of w_{n-1} is the first entry of w_n, and the third entry of w_{n-1} is the second entry of w_n.

d. Could you find the -5^{th} term in the sequence? If so, how? What is its value?

Yes.

$$w_{-5} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{-6} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix}$$

4. Mr. Johnson completed a survey on the number of hours he spends weekly watching different types of television programs. He determined that he spends 30 hours a week watching programs of the following types: comedy, drama, movies, competition, and sports. He spends half as much time watching competition shows as he does watching dramas. His time watching sports is double his time watching dramas. He spends an equal amount of time watching comedies and movies. The total amount of time spent watching comedies and movies is the same as the total amount of time spent watching dramas and competition shows.

   Write and solve a system of equations to determine how many hours Mr. Johnson watches each type of programming each week.

   Let f = hours watching comedy, d = hours watching drama, m = hours watching movies, c = hours watching competition shows, and s = hours watching sports.

$$f + d + m + c + s = 30$$
$$0.5d = c$$
$$s = 2d$$
$$f = m$$
$$f + m = d + c$$
$$f = 4.5, d = 6, m = 4.5, c = 3, s = 12$$
$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0.5 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 \end{vmatrix} \begin{vmatrix} f \\ d \\ m \\ c \\ s \end{vmatrix} = \begin{vmatrix} 30 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

   Mr. Johnson spends 4.5 hours watching comedies, 6 hours watching dramas, 4.5 hours watching movies, 3 hours watching competition shows, and 12 hours watching sports each week.

5. A copper alloy is a mixture of metals having copper as their main component. Copper alloys do not corrode easily and conduct heat. They are used in all types of applications including cookware and pipes. A scientist is studying different types of copper alloys and has found one containing copper, zinc, tin, aluminum, nickel, and silicon. The alloy weighs 3.2 kilograms. The percentage of aluminum is triple the percentage of zinc. The percentage of silicon is half that of zinc. The percentage of zinc is triple that of nickel. The percentage of copper is fifteen times the sum of the percentages of aluminum and zinc combined. The percentage of copper is nine times the combined percentages of all the other metals.

   a. Write and solve a system of equations to determine the percentage of each metal in the alloy.

   Let c = percentage of copper, z = percentage of zinc, t = percentage of tin, a = percentage of aluminum, n = percentage of nickel, s = percentage of silicon:

$$c+z+t+a+n+s=3.2$$

$$a=3z$$

$$s=0.5z$$

$$z=3n$$

$$c=15(a+z)$$

$$c=9(z+t+a+n+s)$$

$$c=90\%,\ z=1.5\%,\ t=2.75\%,\ a=4.5\%,\ N=0.5\%,\ s=0.75\%$$

   The alloy has 90% copper, 1.5% zinc, 2.75% tin, 4.5% aluminum, 0.5% nickel, and 0.75% silicon.

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -3 & 0 & 1 & 0 & 0 \\ 0 & -0.5 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 & 0 \\ 1 & -15 & 0 & -15 & 0 & 0 \\ 1 & -9 & -9 & -9 & -9 & -9 \end{vmatrix} \begin{vmatrix} c \\ z \\ t \\ a \\ n \\ s \end{vmatrix} = \begin{vmatrix} 3.2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

   b. How many kilograms of each alloy are present in the sample?

   Copper: $0.90(3.2)=2.88$ kg

   Zinc: $0.015(3.2)=0.048$ kg

   Tin: $0.0275(3.2)=0.088$ kg

   Aluminum: $0.045(3.2)=0.144$ kg

   Nickel: $0.005(3.2)=0.016$ kg

   Silicon: $0.0075(3.2)=0.024$ kg