

Lesson 16: Solving General Systems of Linear Equations

Classwork

Example 1

A scientist measured the greatest linear dimension of several irregular metal objects. He then used water displacement to calculate the volume of each of the objects. The data he collected are $(1, 3)$, $(2, 5)$, $(4, 9)$, and $(6, 20)$, where the first coordinate represents the linear measurement of the object in centimeters, and the second coordinate represents the volume in cubic centimeters. Knowing that volume measures generally vary directly with the cubed value of linear measurements, he wants to try to fit this data to a curve in the form of $v(x) = ax^3 + bx^2 + cx + d$.

- Represent the data using a system of equations.
- Represent the system using a matrix equation in the form $Ax = b$.
- Use technology to solve the system.
- Based on your solution to the system, what cubic equation models the data?
- What are some of the limitations of the model?

Exercises 1–3

1. An attendance officer in a small school district noticed a trend among the four elementary schools in the district. This district used an open enrollment policy, which means any student within the district could enroll at any school in the district. Each year, 10% of the students from Adams Elementary enrolled at Davis Elementary, and 10% of the students from Davis enrolled at Adams. In addition, 10% of the students from Brown Elementary enrolled at Carson Elementary, and 20% of the students from Brown enrolled at Davis. At Carson Elementary, about 10% of students enrolled at Brown, and 10% enrolled at Davis, while at Davis, 10% enrolled at Brown, and 20% enrolled at Carson. The officer noted that this year, the enrollment was 490, 250, 300, and 370 at Adams, Brown, Carson, and Davis, respectively.
 - a. Represent the relationship that reflects the annual movement of students among the elementary schools using a matrix.
 - b. Write an expression that could be used to calculate the attendance one year prior to the year cited by the attendance officer. Find the enrollment for that year.
 - c. Assuming that the trend in attendance continues, write an expression that could be used to calculate the enrollment two years after the year cited by the attendance officer. Find the attendance for that year.
 - d. Interpret the results to part (c) in context.

2. Mrs. Kenrick is teaching her class about different types of polynomials. They have just studied quartics, and she has offered 5 bonus points to anyone in the class who can determine the quartic that she has displayed on the board. The quartic has 5 points identified: $(-6, 25)$, $(-3, 1)$, $(-2, \frac{7}{3})$, $(0, -5)$, and $(3, 169)$. Logan really needs those bonus points and remembers that the general form for a quartic is $y = ax^4 + bx^3 + cx^2 + dx + e$. Can you help Logan determine the equation of the quartic?
- Write the system of equations that would represent this quartic.
 - Write a matrix that would represent the coefficients of this quartic.
 - Write an expression that could be used to calculate coefficients of the equation.
 - Explain the answer in the context of this problem.

3. The Fibonacci numbers are the numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, Each number beyond the second is the sum of the previous two.

Let $u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $u_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $u_4 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $u_5 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$, and so on.

a. Show that $u_{n+1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} u_n$.

- b. How could you use matrices to find u_{30} ? Use technology to find u_{30} .

c. If $u_n = \begin{bmatrix} 165580141 \\ 267914296 \end{bmatrix}$, find u_{n-1} . Show your work.

Problem Set

1. The system of equations is given:

$$1.2x + 3y - 5z + 4.2w + v = 0$$

$$6x = 5y + 2w$$

$$3y + 4.5z - 6w + 2v = 10$$

$$9x - y + z + 2v = -3$$

$$-4x + 2y - w + 3v = 1$$

- Represent this system using a matrix equation.
 - Use technology to solve the system. Show your solution process, and round your entries to the tenths place.
2. A caterer was preparing a fruit salad for a party. She decided to use strawberries, blackberries, grapes, bananas, and kiwi. The total weight of the fruit was 10 pounds. Based on guidelines from a recipe, the weight of the grapes was equal to the sum of the weight of the strawberries and blackberries; the total weight of the blackberries and kiwi was 2 pounds; half the total weight of fruit consisted of kiwi, strawberries, and bananas; and the weight of the grapes was twice the weight of the blackberries.
- Write a system of equations to represent the constraints placed on the caterer when she made the fruit salad. Be sure to define your variables.
 - Represent the system using a matrix equation.
 - Solve the system using the matrix equation. Explain your solution in context.
 - How helpful would the solution to this problem likely be to the caterer as she prepares to buy the fruit?
3. Consider the sequence 1, 1, 1, 3, 5, 9, 17, 31, 57, ... where each number beyond the third is the sum of the previous three. Let w_n be the points with the n^{th} , $(n + 1)^{\text{th}}$, and $(n + 2)^{\text{th}}$ terms of the sequence.
- Find a 3×3 matrix A so that $Aw_n = w_{n+1}$ for each n .
 - What is the 30th term of the sequence?
 - What is A^{-1} ? Explain what A^{-1} represents in terms of the sequence. In other words, how can you find w_{n-1} if you know w_n ?
 - Could you find the -5^{th} term in the sequence? If so, how? What is its value?
4. Mr. Johnson completed a survey on the number of hours he spends weekly watching different types of television programs. He determined that he spends 30 hours a week watching programs of the following types: comedy, drama, movies, competition, and sports. He spends half as much time watching competition shows as he does watching dramas. His time watching sports is double his time watching dramas. He spends an equal amount of time watching comedies and movies. The total amount of time spent watching comedies and movies is the same as the total amount of time spent watching dramas and competition shows.
- Write and solve a system of equations to determine how many hours Mr. Johnson watches each type of programming each week.

5. A copper alloy is a mixture of metals having copper as their main component. Copper alloys do not corrode easily and conduct heat. They are used in all types of applications including cookware and pipes. A scientist is studying different types of copper alloys and has found one containing copper, zinc, tin, aluminum, nickel, and silicon. The alloy weighs 3.2 kilograms. The percentage of aluminum is triple the percentage of zinc. The percentage of silicon is half that of zinc. The percentage of zinc is triple that of nickel. The percentage of copper is fifteen times the sum of the percentages of aluminum and zinc combined. The percentage of copper is nine times the combined percentages of all the other metals.
- Write and solve a system of equations to determine the percentage of each metal in the alloy.
 - How many kilograms of each alloy are present in the sample?