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Lesson 11: Matrix Addition Is Commutative

Student Outcomes

* Students prove both geometrically and algebraically that matrix addition is commutative.

Lesson Notes

In Topic A, we interpreted matrices as representing network diagrams. In that context, an arithmetic system for matrices was natural. Given two matrices and of equal dimensions, we defined the matrix product and the matrix sum . Both of these operations had a meaning within the context of networks. In Topic B, we returned to our interpretation of matrices as representing the geometric effect of linear transformations from Module 1. We have found that the matrix product has meaning in this context; it is the composition of transformations. In this lesson, we will explore the question, “Can we give matrix addition meaning in the setting of geometric transformations?”

Classwork

Opening Exercise (5 minutes)

Allow students time to work on the Opening Exercise independently before discussing as a class.

Opening Exercise

**MP.3**

Kiamba thinks for all matrices. Rachel thinks it is not always true. Who is correct? Explain.

*Scaffolding:*

Provide students with concrete examples if necessary.

and

Let and .

What is the sum of ?

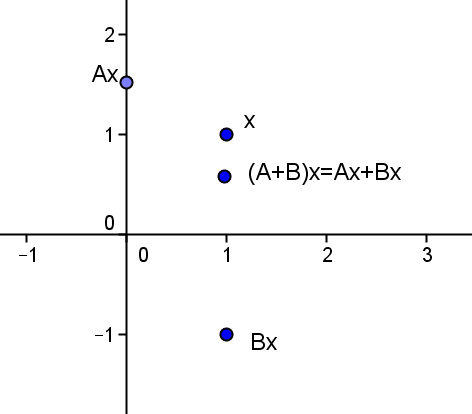
The two matrices must be equal because each of the sums must be equal according to the commutative property of addition of real numbers. Kiamba is correct.

* Can we say that matrix addition is commutative?
  + *Yes. The order in which we add the matrices does not change the sum.*
* Will this hold true if we change the dimensions of the matrices being added?
  + *Yes. Regardless of the size of the matrices, the two sums ( and ) would still be the same. [Note: Demonstrate with two matrices if students seem unsure.]*
* So we see that matrix addition is commutative, but we still have not determined the geometric meaning of matrix addition.

Exercise 1 (12 minutes)

Allow students time to work in groups on Exercise 1. Optionally, give students colored pencils or a transparency and fine-point dry erase marker to mark their points.

Exercises 1–6

1. In two-dimensional space, let be the matrix representing a rotation about the origin through an angle of , and let be the matrix representing a reflection about the -axis. Let be the point .
   1. Write down the matrices,,and

* 1. Write down the image points of , and , and plot them on graph paper.

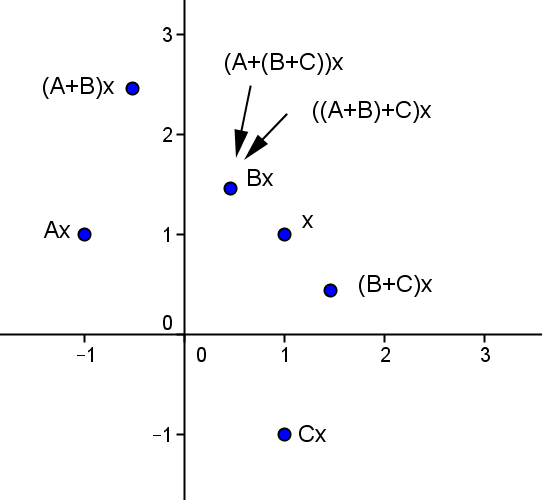
* 1. What do you notice about compared to and ?

The point is equal to the sum of the points and by the distributive property.

* Since , does it follow that ?
  + *Yes, and must both map to the same point since we know that the addition of points is commutative. Since maps to the point by the distributive property*, must also map to the point since .
* What does this say about matrix addition?
  + *It confirms what we saw in the opening exercise. Matrix addition is commutative.*

Exercise 2 (13 minutes)

Allow students time to work in groups on Exercise 2 before discussing as a class.

1. For three matrices of equal size, ,,anddoes it follow that
   1. Determine if the statement is true geometrically. Let be the matrix representing a reflection across the -axis. Let be the matrix representing a counterclockwise rotation of . Let be the matrix representing a reflection about the -axis. Let be the point .

From the graph, we see that .

* 1. Confirm your results algebraically.
  2. What do your results say about matrix addition?

Matrix addition is associative.

* What does this exercise tell us about matrix addition?
  + *It is associative.*
* How did the graph support the idea that matrix addition is associative?

**MP.2**

* + *and mapped to the same point.*
* Would this still be true if we used matrices of different (but still equal) dimensions?
  + *Yes. The addition would still be associative. For example, we could do the same type of operations using matrices, but geometrically it would represent points in 3-D space rather than on a coordinate plane.*
* Could we say ?
  + *Yes. We just demonstrated that* . We can then say that  
    because we know that matrix addition is commutative.

Exercises 3–6 (5 minutes)

Allow students time to work in groups on Exercises 3–6 before discussing as a class.

1. If , what are the coordinates of a point with the property is the origin ?
2. Suppose , and matrix has the property that is the origin. What is the matrix
3. For three matrices of equal size, ,,andwhere represents a reflection across the line , represents a counterclockwise rotation of , represents a reflection across the -axis, and
   1. Show that matrix addition is commutative: .
   2. Show that matrix addition is associative: .

*Scaffolding:*

Provide early finishers with this challenge question.

If for all , must every entry of be ?

1. Let and be matrices of the same dimensions. Use the commutative property of addition of two matrices to prove .

If we treat as one matrix, then

Closing (5 minutes)

Discuss the key points of the lesson as a class.

* What can be said about matrix addition?
  + *Matrix addition, like addition of numbers, is both commutative and associative.*
* What is the geometric meaning of matrix addition? Illustrate with an example.
  + *maps to the same point as .*
  + *Answers may vary. ,,and. ,,,,andTherefore,  
    .*

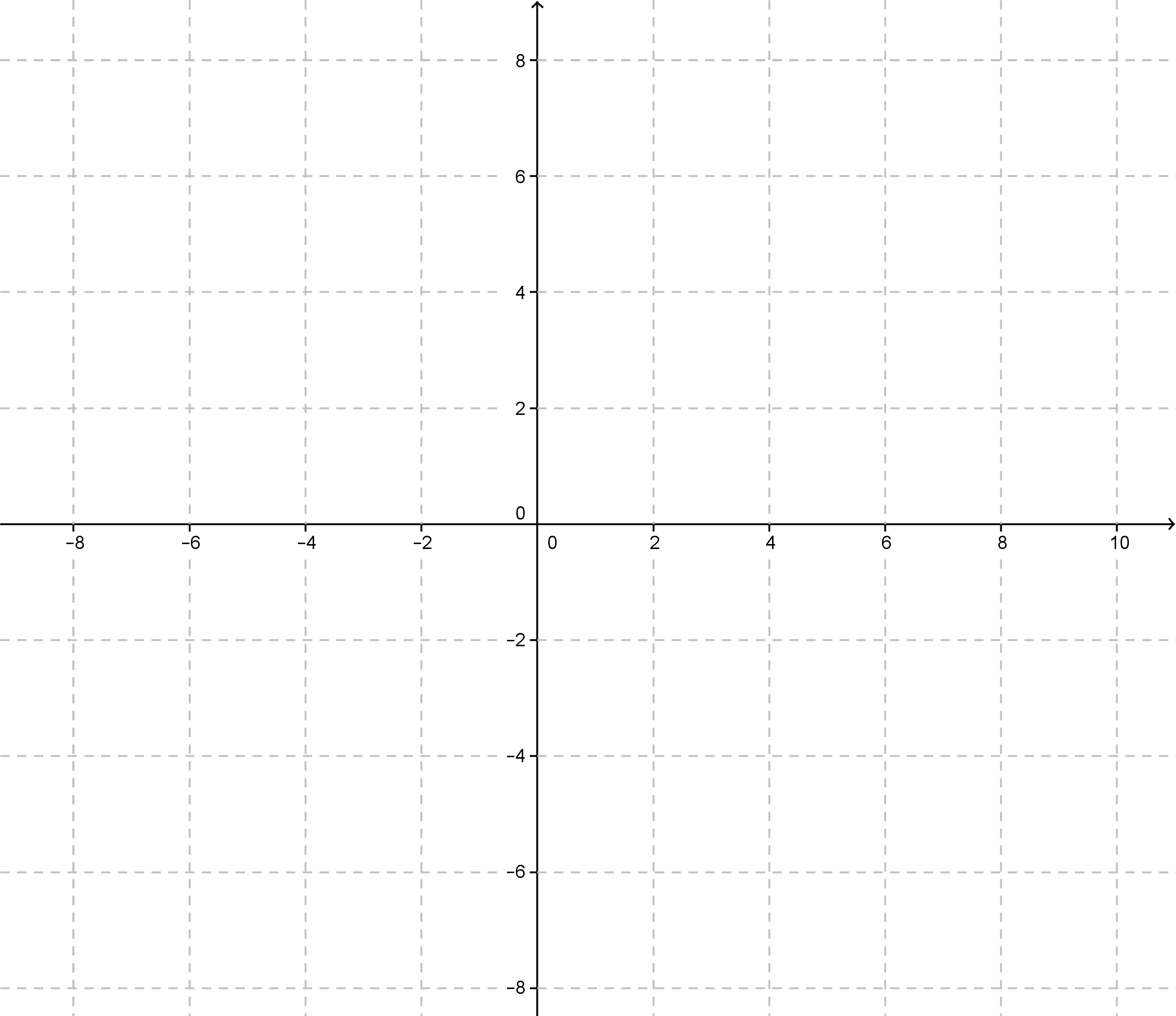
Exit Ticket (5 minutes)

Name Date

Lesson 11: Matrix Addition Is Commutative

Exit Ticket

1. Let , and.
   1. Find and plot the points *,* , and on the axes below.

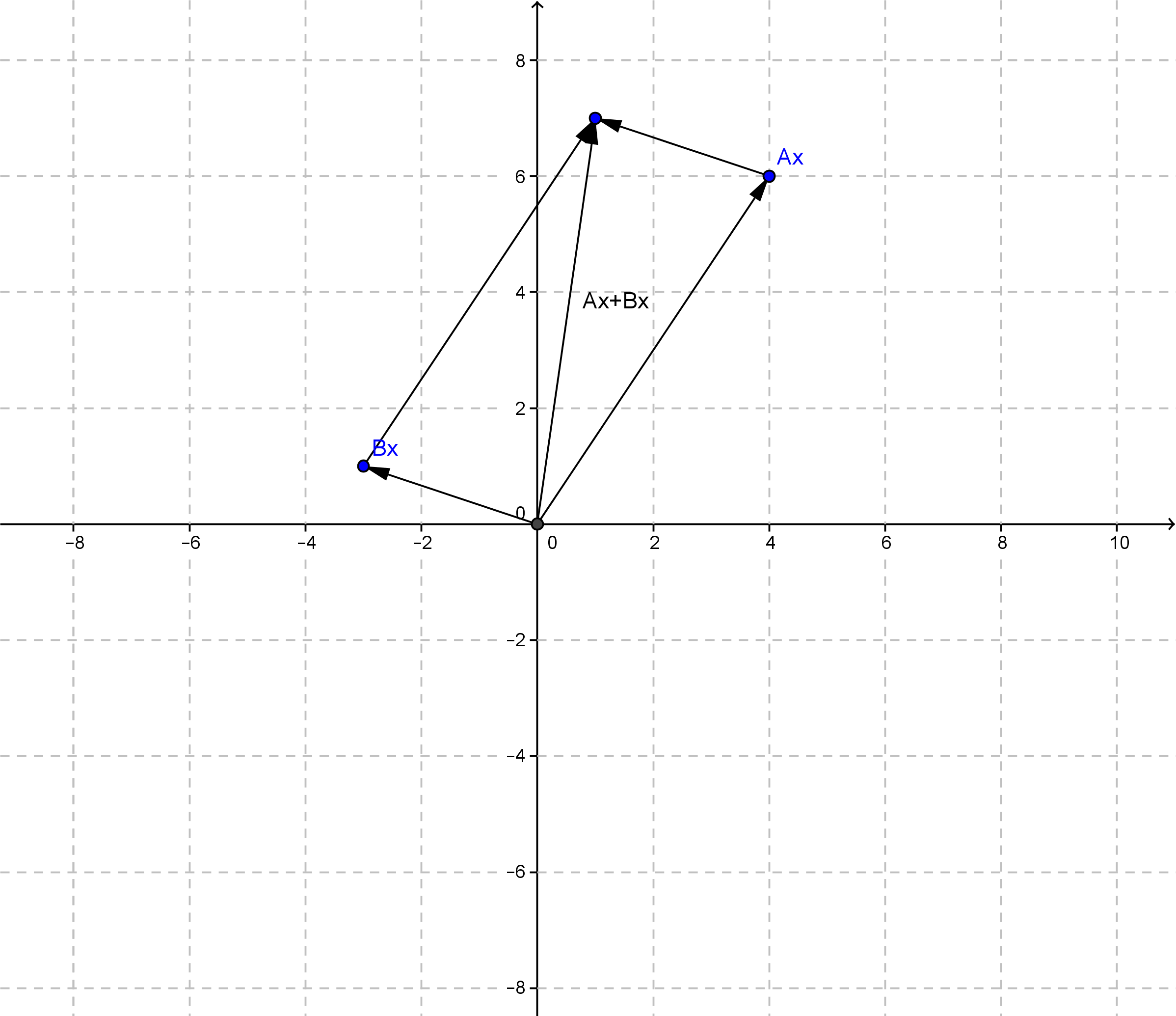


* 1. Show algebraically that matrix addition is commutative, .

1. Let and. Prove *.*

Exit Ticket Sample Solutions

1. Let , and.
   1. Find and plot the points *,* , and on the axes below.



* 1. Show algebraically that matrix addition is commutative, .

,,and..

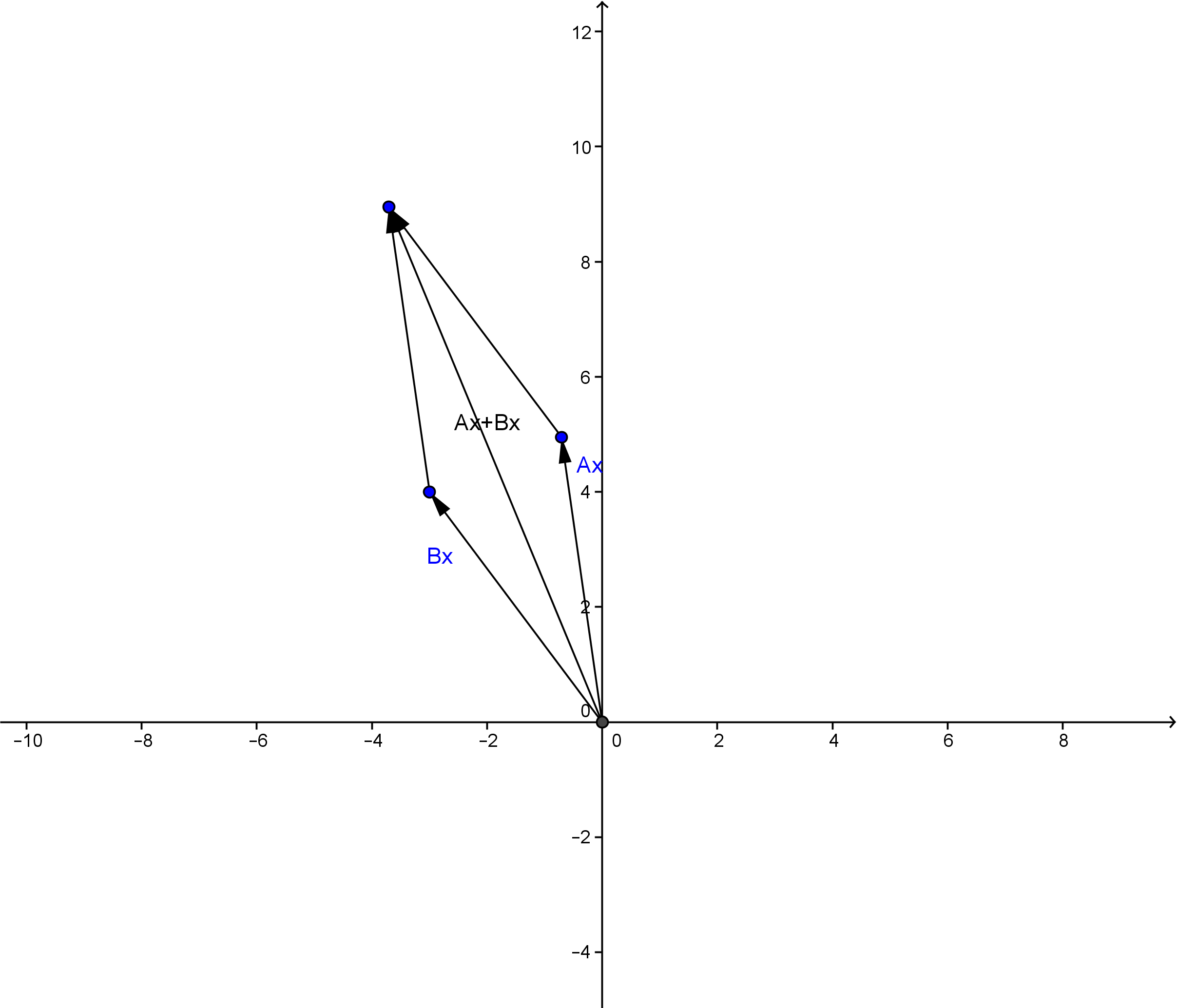
1. Let and. Prove .

Problem Set Sample Solutions

1. Let be matrix transformation representing a rotation of about the origin and be a reflection across the   
   -axis. Let .
   1. Represent and as matrices, and find
   2. Represent and as matrices, and find .
   3. Graph your answer to part (b).

See graph in part (d).

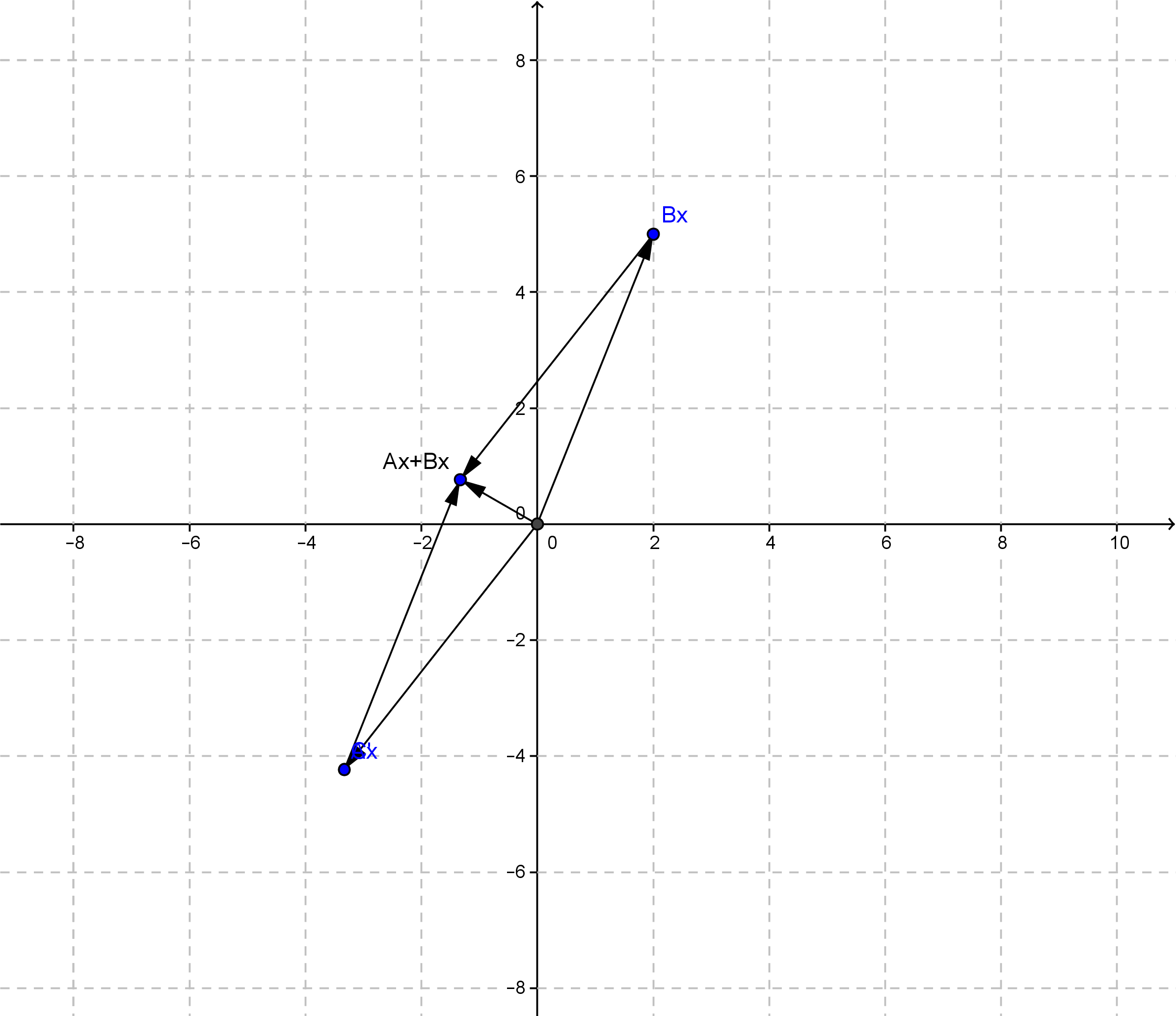
* 1. Draw the parallelogram containing , , and .



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   -axis. Let .
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   2. Represent and as matrices, and find .
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See graph in part (d).

* 1. Draw the parallelogram containing , , and .



1. Let and be matrices of the same dimensions.
   1. Use the associative property of addition for three matrices to prove .

If we treat as one matrix, then

* 1. Make an argument for the associative and commutative properties of addition of matrices to be true for finitely many matrices being added.

For finitely many matrices, we can always use the formula that has been proven already and break the rest of the problem down into that many pieces, just like we did in part (a). Since this is true for any finite number, we could also use the formula that has been proven for one less than whatever number we are trying to prove the property true for.

1. Let be an matrix with element in the th row, th column , and be an matrix with element in the th row, th column . Present an argument that .

The entry in the th row, th column of will be , which is equal to , which is the entry in the th row, th column of . Since this is true for any element of , we have that for any two matrices of equal dimensions.

1. For integers , define , read “ plus ” where is defined normally.
   1. Is this form of addition commutative? Explain why or why not.

Yes. .

* 1. Is this form of addition associative? Explain why or why not.

No.

1. For integers , define .
   1. Is this form of addition commutative? Explain why or why not.

No. , and .

* 1. Is this form of addition associative? Explain why or why not.

Yes. , and .