Lesson 11: Matrix Addition Is Commutative

Classwork

Opening Exercise

Kiamba thinks for all matrices. Rachel thinks it is not always true. Who is correct? Explain.

Exercises 1–6

1. In two-dimensional space, let be the matrix representing a rotation about the origin through an angle of , and let be the matrix representing a reflection about the -axis. Let be the point .
   1. Write down the matrices,,and
   2. Write down the image points of , and , and plot them on graph paper.
   3. What do you notice about compared to and ?
2. For three matrices of equal size, anddoes it follow that
   1. Determine if the statement is true geometrically. Let be the matrix representing a reflection across the   
      -axis. Let be the matrix representing a counterclockwise rotation of . Let be the matrix representing a reflection about the -axis. Let be the point .
   2. Confirm your results algebraically.
   3. What do your results say about matrix addition?
3. If , what are the coordinates of a point with the property is the origin ?
4. Suppose , and matrix has the property that is the origin. What is the matrix
5. For three matrices of equal size, andwhere represents a reflection across the line , represents a counterclockwise rotation of represents a reflection across the -axis, and
   1. Show that matrix addition is commutative: .
   2. Show that matrix addition is associative: .
6. Let and be matrices of the same dimensions. Use the commutative property of addition of two matrices to prove .

Problem Set

1. Let be matrix transformation representing a rotation of about the origin and be a reflection across the   
   -axis. Let .
   1. Represent and as matrices, and find
   2. Represent and as matrices, and find .
   3. Graph your answer to part (b).
   4. Draw the parallelogram containing , , and .
2. Let be matrix transformation representing a rotation of about the origin and be a reflection across the  
    -axis. Let .
   1. Represent and as matrices, and find
   2. Represent and as matrices, and find .
   3. Graph your answer to part (b).
   4. Draw the parallelogram containing , and .
3. Let and be matrices of the same dimensions.
   1. Use the associative property of addition for three matrices to prove .
   2. Make an argument for the associative and commutative properties of addition of matrices to be true for finitely many matrices being added.
4. Let be an matrix with element in the th row, th column , and be an matrix with element in the th row, th column . Present an argument that .
5. For integers , define , read “ plus ” where is defined normally.
   1. Is this form of addition commutative? Explain why or why not.
   2. Is this form of addition associative? Explain why or why not.
6. For integers , define .
   1. Is this form of addition commutative? Explain why or why not.
   2. Is this form of addition associative? Explain why or why not.