Lesson 11: Matrix Addition Is Commutative

Classwork

Opening Exercise

Kiamba thinks $A+B=B+A$ for all $2×2$ matrices. Rachel thinks it is not always true. Who is correct? Explain.

Exercises 1–6

1. In two-dimensional space, let $A$ be the matrix representing a rotation about the origin through an angle of $45°$, and let $B$ be the matrix representing a reflection about the $x$-axis. Let $x$ be the point $\left(\begin{matrix}1\\1\end{matrix}\right)$.
	1. Write down the matrices$ A$,$ B$,$ $and $A+B.$
	2. Write down the image points of $Ax, Bx$, and $\left(A+B\right)x$, and plot them on graph paper.
	3. What do you notice about $\left(A+B\right)x$ compared to $Ax$ and $Bx$?
2. For three matrices of equal size, $A, B, $and$ C, $does it follow that$A+\left(B+C\right)=\left(A+B\right)+C? $
	1. Determine if the statement is true geometrically. Let $A$ be the matrix representing a reflection across the
	$y$-axis. Let $B$ be the matrix representing a counterclockwise rotation of $30°$. Let $C$ be the matrix representing a reflection about the $x$-axis. Let $x$ be the point $\left(\begin{matrix}1\\1\end{matrix}\right)$.
	2. Confirm your results algebraically.
	3. What do your results say about matrix addition?
3. If $x=\left(\begin{matrix}x\\y\\z\end{matrix}\right)$, what are the coordinates of a point $y$ with the property $x+y$ is the origin $O=\left(\begin{matrix}0\\0\\0\end{matrix}\right)$?
4. Suppose $A=\left(\begin{matrix}11&-5&2\\-34&6&19\\8&-542&0\end{matrix}\right)$, and matrix $B$ has the property that $Ax+Bx$ is the origin. What is the matrix $B?$
5. For three matrices of equal size, $A, B, $and$ C, $where $A$ represents a reflection across the line $y=x$, $B$ represents a counterclockwise rotation of $45°,$ $C$ represents a reflection across the $y$-axis, and $x=\left(\begin{matrix}1\\2\end{matrix}\right):$
	1. Show that matrix addition is commutative: $Ax+Bx=Bx+Cx$.
	2. Show that matrix addition is associative: $Ax+\left(Bx+Cx\right)=\left(Ax+Bx\right)+Cx$.
6. Let $A,B,C,$ and$ D$ be matrices of the same dimensions. Use the commutative property of addition of two matrices to prove $A+B+C=C+B+A$.

Problem Set

1. Let $A$ be matrix transformation representing a rotation of $45°$ about the origin and $B$ be a reflection across the
$y$-axis. Let $x=(3,4)$.
	1. Represent $A $and$ B$ as matrices, and find $A+B.$
	2. Represent $Ax $and $Bx$ as matrices, and find $(A+B)x$.
	3. Graph your answer to part (b).
	4. Draw the parallelogram containing $Ax$, $Bx$, and $(A+B)x$.
2. Let $A$ be matrix transformation representing a rotation of $300°$ about the origin and $B$ be a reflection across the
 $x$-axis. Let $x=(2,-5)$.
	1. Represent $A $and$ B$ as matrices, and find $A+B.$
	2. Represent $Ax $and $Bx$ as matrices, and find $(A+B)x$.
	3. Graph your answer to part (b).
	4. Draw the parallelogram containing $Ax$, $Bx,$ and $(A+B)x$.
3. Let $A,B,C, $and$ D$ be matrices of the same dimensions.
	1. Use the associative property of addition for three matrices to prove $\left(A+B\right)+\left(C+D\right)=A+\left(B+C\right)+D$.
	2. Make an argument for the associative and commutative properties of addition of matrices to be true for finitely many matrices being added.
4. Let $A$ be an $m×n$ matrix with element in the $i$th row, $j$th column $a\_{ij}$, and $B$ be an $m×n$ matrix with element in the $i$th row, $j$th column $b\_{ij}$. Present an argument that $A+B=B+A$.
5. For integers $x, y$, define $x⊕y=x⋅y+1$, read “$x$ plus $y$” where $x⋅y$ is defined normally.
	1. Is this form of addition commutative? Explain why or why not.
	2. Is this form of addition associative? Explain why or why not.
6. For integers $x, y$, define $x⊕y=x$.
	1. Is this form of addition commutative? Explain why or why not.
	2. Is this form of addition associative? Explain why or why not.