Lesson 8: Composition of Linear Transformations

Classwork

Opening Exercise

Compute the product for the following pairs of matrices.

1. ,
2. ,
3. ,
4. ,
5. ,

Exploratory Challenge

1. For each pair of matrices and given below:
2. Describe the geometric effect of the transformation .
3. Describe the geometric effect of the transformation .
4. Draw the image of the unit square under the transformation .
5. Draw the image of the transformed square under the transformation .
6. Describe the geometric effect on the unit square of performing first then .
7. Compute the matrix product .
8. Describe the geometric effect of the transformation .
   1. ,
   2. *,*
   3. *,*
   4. *,*
   5. *,*
9. Make a conjecture about the geometric effect of the linear transformation produced by the matrix . Justify your answer.

Lesson Summary

The linear transformation produced by matrix has the same geometric effect as the sequence of the linear transformation produced by matrix followed by the linear transformation produced by matrix .

That is, if matrices and produce linear transformations and in the plane, respectively, then the linear transformation produced by the matrix satisfies  
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Problem Set

1. Let be the matrix representing a dilation of , and let be the matrix representing a reflection across the -axis.
   1. Write and .
   2. Evaluate . What does this matrix represent?
   3. Let and . Find and z.
2. Let be the matrix representing a rotation of , and let be the matrix representing a dilation of .
   1. Write and .
   2. Evaluate . What does this matrix represent?
   3. Let . Find .
3. Let be the matrix representing a dilation of , and let be the matrix representing a reflection across the line   
   .
   1. Write and .
   2. Evaluate . What does this matrix represent?
   3. Let . Find .
4. Let .
   1. Evaluate .
   2. Let . Find .
   3. Graph and .
5. Let .
   1. Evaluate .
   2. Let . Find .
   3. Graph and .
6. Let .
   1. Evaluate .
   2. Let . Find .
   3. Graph and .
7. Let ,, be matrices representing linear transformations.
   1. What does represent?
   2. Will the pattern established in part (a) be true no matter how many matrices are multiplied on the left?
   3. Does represent something different from ? Explain.
8. Let represent any composition of linear transformations in . What is the value of where ?