Lesson 6: Linear Transformations as Matrices

Classwork

Opening Exercise

Let $A=\left(\begin{matrix}7&-2\\5&-3\end{matrix}\right)$,$ x=\left(\begin{matrix}x\_{1}\\x\_{2}\end{matrix}\right)$, and$ y=\left(\begin{matrix}y\_{1}\\y\_{2}\end{matrix}\right)$. Does this represent a linear transformation? Explain how you know.

Exploratory Challenge 1: The Geometry of 3D Matrix Transformations

* 1. What matrix in $R^{2}$ serves the role of $1$ in the real number system? What is that role?
	2. What matrix in $R^{2}$ serves the role of $0$ in the real number system? What is that role?
	3. What is the result of scalar multiplication in $R^{2}$?
	4. Given a complex number $a+bi$, what represents the transformation of that point across the real axis?

Exploratory Challenge 2: Properties of Vector Arithmetic

* 1. Is vector addition commutative? That is, does $x+y=y+x$ for each pair of points in $R^{2}$? What about points in $R^{3}$?
	2. Is vector addition associative? That is, does $\left(x+y\right)+r=x+(y+r)$ for any three points in $R^{2}$? What about points in $R^{3}$?
	3. Does the distributive property apply to vector arithmetic? That is, does $k∙\left(x+y\right)=kx+ky$ for each pair of points in $R^{2}$? What about points in $R^{3}$?
	4. Is there an identity element for vector addition? That is, can you find a point $a$ in $R^{2}$ such that $x+a=x$ for every point $x$ in $R^{2}$? What about for$R^{3}$?
	5. Does each element in $R^{2}$ have an additive inverse? That is, if you take a point $a$ in $R^{2}$, can you find a second point $b$ such that $a+b=0$?

Problem Set

1. Show that the associative property, $x+\left(y+z\right)=\left(x+y\right)+z$*,* holds for the following.
	1. $x=\left(\begin{matrix}3\\-2\end{matrix}\right)$,$y=\left(\begin{matrix}-4\\2\end{matrix}\right)$,$z=\left(\begin{matrix}-1\\5\end{matrix}\right) $
	2. $x=\left(\begin{matrix}2\\-2\\1\end{matrix}\right)$,$y=\left(\begin{matrix}0\\5\\-2\end{matrix}\right)$,$ z=\left(\begin{matrix}3\\0\\-3\end{matrix}\right)$
2. Show that the distributive property, $ k \left(x+y\right)=kx+ky$, holds for the following.
	1. $x=\left(\begin{matrix}5\\-3\end{matrix}\right)$,$y=\left(\begin{matrix}-2\\4\end{matrix}\right)$,$ k=-2$
	2. $x=\left(\begin{matrix}3\\-2\\5\end{matrix}\right)$,$y=\left(\begin{matrix}-4\\6\\-7\end{matrix}\right)$,$ k=-3$
3. Compute the following.
	1. $\left(\begin{matrix}1&0&2\\0&1&2\\2&2&3\end{matrix}\right)\left(\begin{matrix}2\\1\\3\end{matrix}\right)$
	2. $\left(\begin{matrix}-1&2&3\\3&1&-2\\1&-2&3\end{matrix}\right)\left(\begin{matrix}1\\-2\\1\end{matrix}\right)$
	3. $\left(\begin{matrix}1&0&2\\0&1&0\\2&0&3\end{matrix}\right)\left(\begin{matrix}2\\1\\2\end{matrix}\right)$
4. Let $x=\left(\begin{matrix}3\\1\\2\end{matrix}\right)$. Compute $L\left(x\right)=\left[\begin{matrix}a&d&g\\b&e&h\\c&f&i\end{matrix}\right]∙x$, plot the points, and describe the geometric effect to $x$.
	1. $\left[\begin{matrix}-1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$
	2. $\left[\begin{matrix}2&0&0\\0&2&0\\0&0&2\end{matrix}\right]$
	3. $\left[\begin{matrix}0&1&0\\1&0&0\\0&0&1\end{matrix}\right]$
	4. $\left[\begin{matrix}1&0&0\\0&0&1\\0&1&0\end{matrix}\right]$
5. Let $x=\left(\begin{matrix}3\\1\\2\end{matrix}\right)$. Compute $L\left(x\right)=\left[\begin{matrix}a&d&g\\b&e&h\\c&f&i\end{matrix}\right]∙x$. Describe the geometric effect to $x$.
	1. $\left[\begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix}\right]$
	2. $\left[\begin{matrix}3&0&0\\0&3&0\\0&0&3\end{matrix}\right]$
	3. $\left[\begin{matrix}-1&0&0\\0&-1&0\\0&0&-1\end{matrix}\right]$
	4. $\left[\begin{matrix}-1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$
	5. $\left[\begin{matrix}1&0&0\\0&-1&0\\0&0&1\end{matrix}\right]$
	6. $\left[\begin{matrix}1&0&0\\0&1&0\\0&0&-1\end{matrix}\right]$
	7. $\left[\begin{matrix}0&1&0\\1&0&0\\0&0&1\end{matrix}\right]$
	8. $\left[\begin{matrix}1&0&0\\0&0&1\\0&1&0\end{matrix}\right]$
	9. $\left[\begin{matrix}0&0&1\\0&1&0\\1&0&0\end{matrix}\right]$
6. Find the matrix that will transform the point $x=\left(\begin{matrix}1\\3\\2\end{matrix}\right)$ to the following point:
	1. $\left(\begin{matrix}-4\\-12\\-8\end{matrix}\right)$
	2. $\left(\begin{matrix}3\\1\\2\end{matrix}\right)$
7. Find the matrix/matrices that will transform the point $x=\left(\begin{matrix}2\\3\\1\end{matrix}\right)$ to the following point:

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| --- | --- |
| * 1. $x^{'}=\left(\begin{matrix}6\\4\\2\end{matrix}\right)$
 | * 1. $x^{'}=\left(\begin{matrix}-1\\3\\2\end{matrix}\right)$
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