

Lesson 5: Coordinates of Points in Space

Classwork

Opening Exercise

Compute:

a. $(-10 + 9i) + (7 - 5i)$

b. $5 \cdot (2 + 3i)$

c. $\begin{pmatrix} 5 \\ -6 \end{pmatrix} + \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

d. $-2 \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

Exercises

1. Let $\mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Compute $\mathbf{z} = \mathbf{x} + \mathbf{y}$, and draw the associated parallelogram.

2. Let $\mathbf{x} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Compute $\mathbf{z} = \mathbf{x} + \mathbf{y}$, and draw the associated parallelogram.

3. Let $\mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$. Compute $\mathbf{z} = \mathbf{x} + \mathbf{y}$, and draw the associated parallelogram.

4. Let $\mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Compute $\mathbf{z} = 2\mathbf{x}$, and plot \mathbf{x} and \mathbf{z} in the plane.

5. Let $\mathbf{x} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$. Compute $\mathbf{z} = \frac{1}{3}\mathbf{x}$, and plot \mathbf{x} and \mathbf{z} in the plane.

6. Let $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Compute $\mathbf{z} = -3\mathbf{x}$, and plot \mathbf{x} and \mathbf{z} in the plane.

7. Let $\mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$. Compute $\mathbf{z} = \mathbf{x} + \mathbf{y}$, and then plot each of these three points.

8. Let $\mathbf{x} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$. Compute $\mathbf{z} = \mathbf{x} + \mathbf{y}$, and then plot each of these three points.

9. Let $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Compute $\mathbf{z} = 4\mathbf{x}$, and then plot each of the three points.

10. Let $\mathbf{x} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$. Compute $\mathbf{z} = -\frac{1}{2}\mathbf{x}$, and then plot each of the three points. Describe what you see.

Problem Set

1. Find the sum of the following complex numbers, and graph them on the complex plane. Trace the parallelogram that is formed by those two complex numbers, the resultant, and the origin. Describe the geometric interpretation.
 - a. $\mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
 - b. $\mathbf{x} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$
 - c. $\mathbf{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$
 - d. $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
2. Simplify and graph the complex number and the resultant. Describe the geometric effect on the complex number.
 - a. $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $k = 2$, $k\mathbf{x} = ?$
 - b. $\mathbf{x} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$, $k = -\frac{1}{3}$, $k\mathbf{x} = ?$
 - c. $\mathbf{x} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $k = 0$, $k\mathbf{x} = ?$
3. Find the sum of the following points, graph the points and the resultant on a 3-dimensional coordinate plane, and describe the geometric interpretation.
 - a. $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.
 - b. $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$.
 - c. $\mathbf{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$.
4. Simplify the following, graph the point and the resultant on a 3-dimensional coordinate plane, and describe the geometric effect.
 - a. $\mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, $k = 2$, $k\mathbf{x} = ?$
 - b. $\mathbf{x} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$, $k = -1$, $k\mathbf{x} = ?$

5. Find
- Any two different points whose sum is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
 - Any two different points in 3 dimensions whose sum is $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.
 - Any two different complex numbers and their sum will create the degenerate parallelogram.
 - Any two different points in 3 dimensions that their sum lie on the same line.
 - A point that is mapped to $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ after multiplying -2 .
 - A point that is mapped to $\begin{pmatrix} \frac{1}{2} \\ -2 \\ 4 \end{pmatrix}$ after multiplying $-\frac{2}{3}$.
6. Given $\mathbf{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$
- Find $\mathbf{x} + \mathbf{y}$ and graph parallelogram that is formed by \mathbf{x} , \mathbf{y} , $\mathbf{x} + \mathbf{y}$, and the origin.
 - Transform the unit square by multiplying it by the matrix $\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$, and graph the result.
 - What did you find from parts (a) and (b)?
 - What is the area of the parallelogram that is formed by part (a)?
 - What is the determinant of the matrix $\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$?
 - Based on observation, what can you say about the degenerate parallelograms in part (a) and part (b)?
7. We learned that when multiplying -1 to a complex number z , for example $z = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, the resulting complex number $z_1 = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ will be on the same line but on the opposite side of the origin. What matrix will produce the same effect? Verify your answer.
8. A point $z = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$ is transformed to $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$. The final step of the transformation is adding the complex number $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$. Describe a possible transformation that can get this result.