

Lesson 4: Linear Transformations Review

Classwork

Exercises 1–2

1. Describe the geometric effect of each mapping.

a. $L(x) = 9 \cdot x$

b. $L(x) = -\frac{1}{2} \cdot x$

2. Write the formula for the mappings described.

- a. A dilation that expands each interval to 5 times its original size.

- b. A collapse of the interval to the number 0.

Problem Set

1. Suppose you have a linear transformation $L: \mathbb{R} \rightarrow \mathbb{R}$, where $L(3) = 6$, $L(5) = 10$.
 - a. Use the addition property to find $L(6)$, $L(8)$, $L(10)$, and $L(13)$.
 - b. Use the multiplication property to find $L(15)$, $L(18)$, and $L(30)$.
 - c. Find $L(-3)$, $L(-8)$, and $L(-15)$.
 - d. Find the formula for $L(x)$.
 - e. Draw the graph of the function $L(x)$.
2. A linear transformation $L: \mathbb{R} \rightarrow \mathbb{R}$ must have the form of $L(x) = ax$ for some real number a . Consider the interval $[-5, 2]$. Describe the geometric effect of the following, and find the new interval.
 - a. $L(x) = 5x$
 - b. $L(x) = -2x$
3. A linear transformation $L: \mathbb{R} \rightarrow \mathbb{R}$ must have the form of $L(x) = ax$ for some real number a . Consider the interval $[-2, 6]$. Write the formula for the mapping described, and find the new interval.
 - a. A reflection over the origin.
 - b. A dilation with a scale of $\sqrt{2}$.
 - c. A reflection over the origin and a dilation with a scale of $\frac{1}{2}$.
 - d. A collapse of the interval to the number 0.
4. In Module 1, we used 2×2 matrices to do transformations on a square, such as a pure rotation, a pure reflection, a pure dilation, and a rotation with a dilation. Now use those matrices to do transformations on this complex number: $z = 2 + i$. For each transformation below, graph your answers.
 - a. A pure dilation with a factor of 2.
 - b. A pure $\frac{\pi}{2}$ radians counterclockwise rotation about the origin.
 - c. A pure π radians counterclockwise rotation about the origin.
 - d. A pure reflection about the real axis.
 - e. A pure reflection about the imaginary axis.
 - f. A pure reflection about the line $y = x$.
 - g. A pure reflection about the line $y = -x$.
5. Wesley noticed that by multiplying the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ by a complex number z produces a pure $\frac{\pi}{2}$ radians counterclockwise rotation, and multiplying by $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ produces a pure dilation with a factor of 2. So, he thinks he can add these two matrices, which will produce $\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$ and will rotate z by $\frac{\pi}{2}$ radians counterclockwise and dilate z with a factor of 2. Is he correct? Explain your reason.

6. In Module 1, we learned that there is not any real number that will satisfy $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$, which is the addition property of linear transformation. However, we discussed that some fixed complex numbers might work. Can you find two pairs of complex numbers that will work? Show your work.
7. Suppose L is a complex-number function that satisfies the dream conditions: $L(z + w) = L(z) + L(w)$ and $L(kz) = kL(z)$ for all complex numbers z, w , and k . Show $L(z) = mz$ for a fixed complex-number m , the only type of complex-number function that satisfies these conditions?
8. For complex numbers, the linear transformation requires $L(x + y) = L(x) + L(y)$, $L(a \cdot x) = a \cdot L(x)$. Prove that in general $L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ is a linear transformation, where $\begin{pmatrix} x \\ y \end{pmatrix}$ represents $z = x + yi$.