Topic B:

Linear Transformations of Planes and Space

N-VM.C.7, N-VM.C.8, N-VM.C.9, N-VM.C.10, N-VM.C.11

Focus Standards:	N-VM.C.7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
	N-VM.C.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
	N-VM.C.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
	N-VM.C.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	N-VM.C.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
Instructional Days:	10	
Lesson 4:	Linear Transformations Review (S)[1]	
Lesson 5:	Coordinates of Points in Space (P)	
Lesson 6:	Linear Transformations as Matrices (E)	
Lesson 7:	Linear Transformations Applied to Cubes (E)	
Lessons 8–9:	Composition of Linear Transformations (E, E)	
Lesson 10:	Matrix Multiplication Is Not Commutative (P)	
Lesson 11:	Matrix Addition Is Commutative (P)	
Lesson 12:	Matrix Multiplication Is Distributive and Associative (P)	
Lesson 13:	Using Matrix Operations for Encryption (M)	

Topic B explores the usefulness of matrices with dimensions higher than 2×2. The concept of a linear transformations from Module 1 is extended to linear transformations in three- (and higher-) dimensional space. In Lessons 4–6, students use what they know about linear transformations performed on real and complex numbers in two dimension and extend that to three dimensional space. They verify the conditions

[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

of linearity in two dimensional space and make conjectures about linear transformations in three dimensional space. Students add matrices and perform scalar multiplication (**N-VM.C.7**), exploring the geometric interpretations for these operations in three dimensions. In Lesson 7, students examine the geometric effects of linear transformations in $\mathbb{R}^3$ induced by various 3×3 matrices on the unit cube. Students explore these transformations and discover the connections between a 3×3 matrix and the geometric effect of the transformation produced by the matrix. The materials support the use of geometry software, such as the freely available GeoGebra, but software is not required. Students extend their knowledge of the multiplicative inverse and that it exists precisely when the determinant of the matrix is non-zero from the area of a unit square in two dimensions to the volume of the unit cube in three-dimensions (**N-VM.C.10**).

In Lesson 8, students explore a sequence of transformations in two dimensions and this is extended in Lesson 9 to three dimensions. Students see a sequence of transformations as represented by multiplication of several matrices and relate this to a composition. In Lessons 8–9, students practice scalar and matrix multiplication extensively, setting the stage for properties of matrices studied in Lessons 10 and 11. In Lesson 10, students discover that matrix multiplication is not commutative and verify this finding algebraically for 2×2 and 3×3 matrices (**N-VM.C.9**). In Lesson 11, students translate points by matrix addition and see that while matrix multiplication is not commutative, matrix addition is commutative. They also write points in two and three dimensions as single column matrices (vectors) and multiply matrices by vectors (**N-VM.C.11**).

The study of matrices continues in Lesson 12 as students discover that matrix multiplication is associative and distributive (**N-VM.C.9**). In Lesson 13, students recap their understanding of matrix operations—matrix product, matrix sum, and scalar multiplication—and properties of matrices by using matrices and matrix operations to discover encrypted codes. The geometric and arithmetic roles of the zero matrix and the identity matrix are explored in Lessons 12 and 13. Students understand that the zero matrix is similar to the role of 0 in the real number system and the identity matrix is similar to 1 (**N-VM.C.10**).

Throughout Topic B, students study matrix operations in two- and three-dimensional space and relate these abstract representations to the transformations they represent (MP.2). Students have opportunities to use computer programs as tools for examining and understanding the geometric effects of transformations produced by matrices on the unit circle (MP.5).