Name ______________________________ Date ______________

Lesson 1: Special Triangles and the Unit Circle

Exit Ticket

1. Evaluate the following trigonometric expressions, and explain how you used the unit circle to determine your answer.

   a. $\sin\left(\pi + \frac{\pi}{3}\right)$

   b. $\cos\left(2\pi - \frac{\pi}{6}\right)$

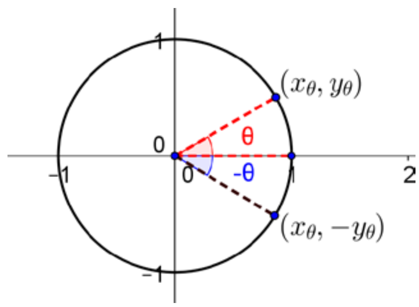
2. Corinne says that for any real number θ, $\cos(\theta) = \cos(\theta - \pi)$. Is she correct? Explain how you know.

Name ______________________________ Date ____________

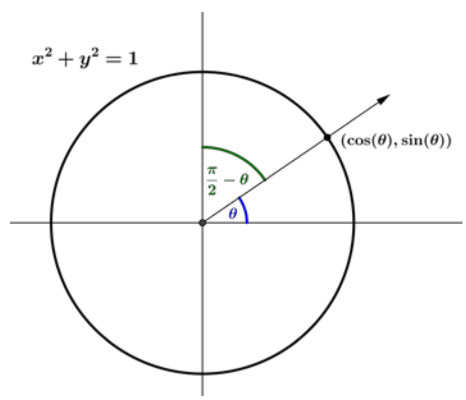
Lesson 2: Properties of Trigonometric Functions

Exit Ticket

1. From the unit circle given, explain why the cosine function is an even function with symmetry about the y-axis, and the sine function is an odd function with symmetry about the origin.



2. Use the unit circle to explain why $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$ for θ as shown in the figure at right.



Name ______________________________ Date ______________

Lesson 3: Addition and Subtraction Formulas

Exit Ticket

1. Prove that $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.

2. Use the addition and subtraction formulas to evaluate the given trigonometric expressions.

   a. $\sin\left(\frac{\pi}{12}\right)$

   b. $\tan\left(\frac{13\pi}{12}\right)$

Name ____ Date ____________________

Lesson 4: Addition and Subtraction Formulas

Exit Ticket

1. Show that $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$.

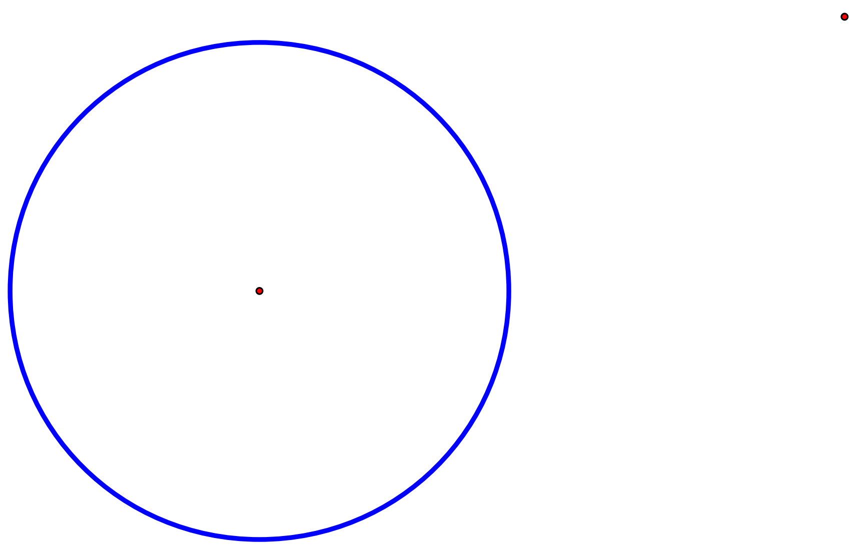
2. Evaluate $\cos\left(\frac{7\pi}{12}\right)$ using the half-angle formula, and then verify your solution using a different formula.

Name ____ Date ____________________

Lesson 5: Tangent Lines and the Tangent Function

Exit Ticket

1. Use a compass and a straightedge to construct the tangent lines to the given circle that pass through the given point.



2. Explain why your construction produces lines that are indeed tangent to the given circle.

Name ____ Date ____________________

Lesson 6: Waves, Sinusoids, and Identities

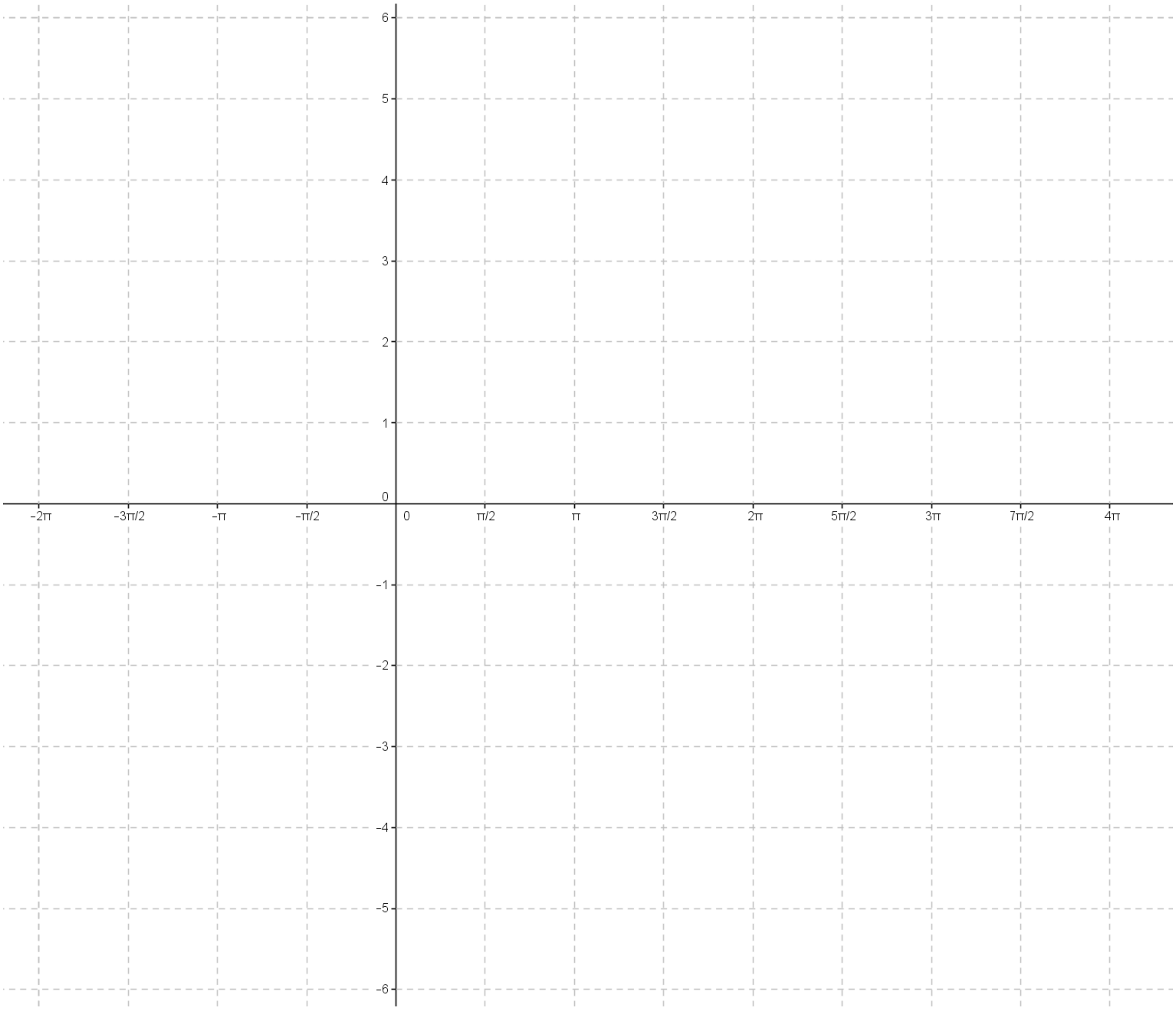
Exit Ticket

1. Use appropriate identities to rewrite the wave equation shown below in the form $h(x) = a\cos(x - c)$.

$$h(x) = 6\sin(x) + 8\cos(x)$$

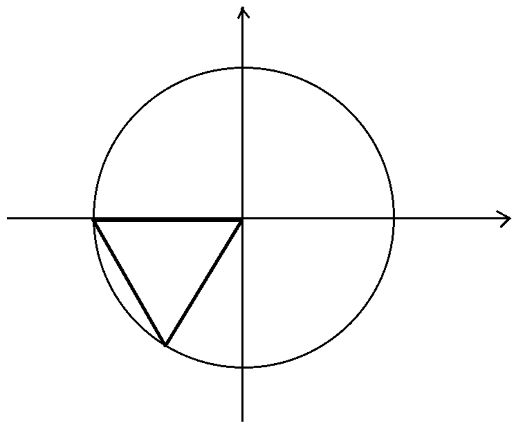
2. Rewrite $h(x) = \sqrt{3}\sin(x) + \cos(x)$ in the form $h(x) = a\cos(x - c)$.

Blank Graph Paper with Trigonometric Scale



Name ____ Date ____________________

1. An equilateral triangle is drawn within the unit circle centered at the origin as shown.



Explain how one can use this diagram to determine the values of $\sin\left(\frac{4\pi}{3}\right)$, $\cos\left(\frac{4\pi}{3}\right)$, and $\tan\left(\frac{4\pi}{3}\right)$.

2. Suppose x is a real number with $0 < x < \frac{\pi}{4}$.

   a. Set $a = \sin(\pi - x)$, $b = \cos(\pi + x)$, $c = \sin(x - \pi)$, and $d = \cos(2\pi - x)$.
   Arrange the values a, b, c, and d in increasing order, and explain how you determined their order.

b. Use the unit circle to explain why $\tan(\pi - x) = -\tan(x)$.

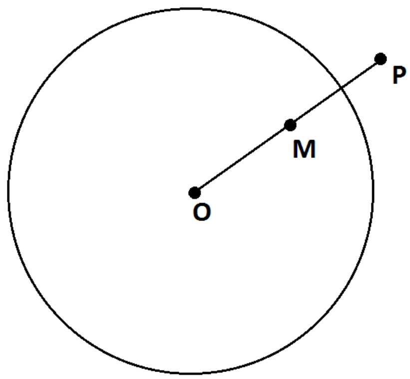
3.

a. Using a diagram of the unit circle centered at the origin, explain why $f(x) = \cos(x)$ is an even function.

b. Using a diagram of the unit circle centered at the origin, explain why $\sin(x - 2\pi) = \sin(x)$ for all real values x.

c. Explain why $\tan(x + \pi) = \tan(x)$ for all real values x.

4. The point P shown lies outside the circle with center O. Point M is the midpoint of the line segment $\overline{OP}$.



a. Use a ruler and compass to construct a line through P that is tangent to the circle.

b. Explain how you know that your construction does indeed produce a tangent line.

5. Each rectangular diagram below contains two pairs of right triangles, each having a hypotenuse of length 1. One pair of triangles has an acute angle measuring x radians. The other pair of triangles has an acute angle measuring y radians.

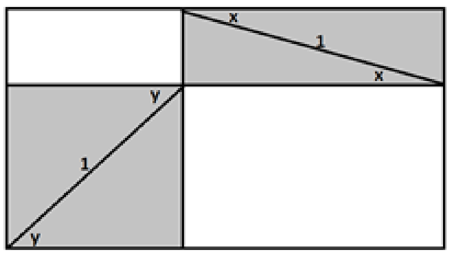


Figure 1

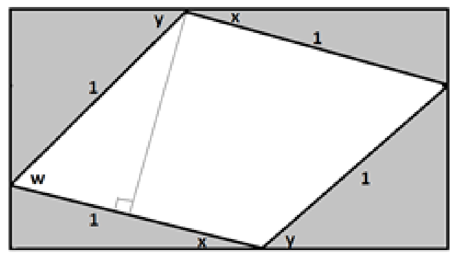


Figure 2

a. Using Figure 1, write an expression, in terms of x and y, for the area of the non-shaded region.

b. Figure 2 contains a quadrilateral which is not shaded and contains angle w. Write an expression, in terms of x and y, for the measure of angle w.

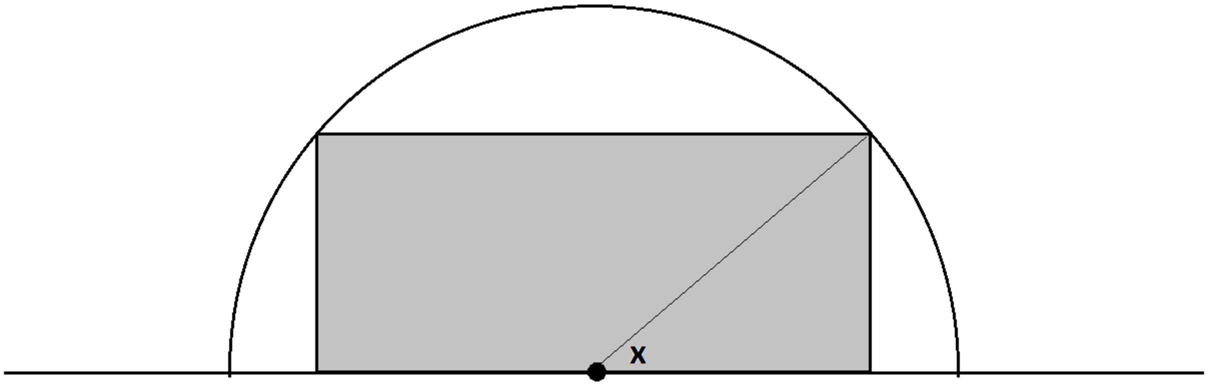
c. Using Figure 2, write an expression, in terms of w, for the non-shaded area. Explain your work.

d. Use the results of parts (a), (b), and (c) to show why $\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$ is a valid formula.

e. Suppose α is a real number between $\frac{\pi}{2}$ and π and y is a real number between 0 and $\frac{\pi}{2}$. Use your result from part (d) to show the following:

$\cos(\alpha + y) = \cos(\alpha)\cos(y) - \sin(\alpha)\sin(y)$.
Explain your work.

6. A rectangle is drawn in a semicircle of radius 3 with its base along the base of the semicircle as shown.



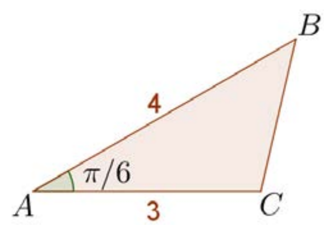
Find, to two decimal places, values for real numbers a and b so that $a\cos(x+b)$ represents the perimeter of the rectangle if the real number x is the measure of the angle shown.

Name ___________________________ Date ______________

Lesson 7: An Area Formula for Triangles

Exit Ticket

1. Find the area of ΔABC.



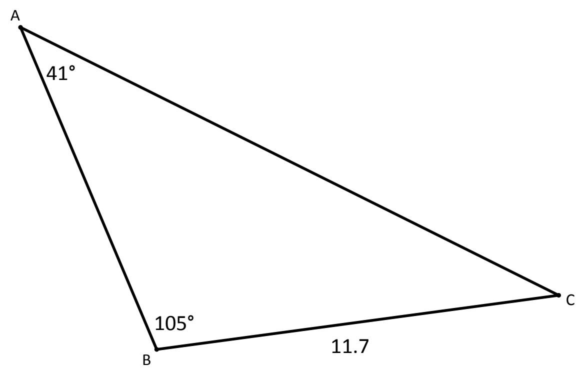
2. Explain why $\frac{1}{2}ab\sin(\theta)$ gives the area of a triangle with sides a and b and included angle θ.

Name ______________________________ Date ____________

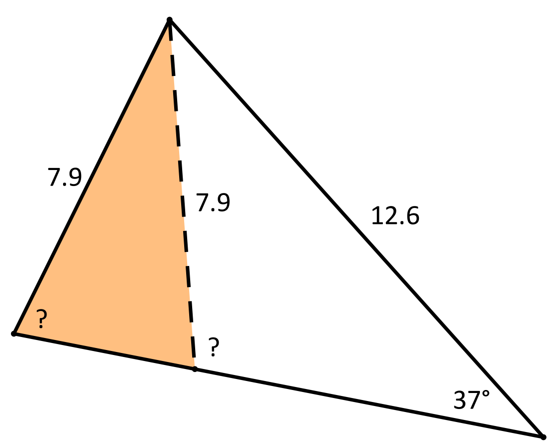
Lesson 8: Law of Sines

Exit Ticket

1. Find the length of side AC in the triangle below.



2. A triangle has sides with lengths 12.6 and 7.9. The angle opposite 7.9 is $37°$. What are the possible values of the measure of the angle opposite 12.6?



Name ____ Date ____________________

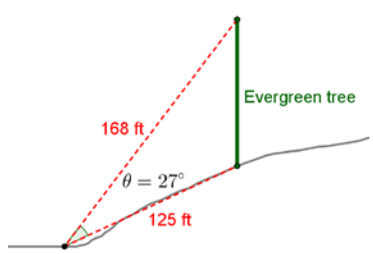
Lesson 9: Law of Cosines

Exit Ticket

Use the law of cosines to solve the following problems.

1. Josie wishes to install a new television that will take up $15°$ of her vertical field of view. She uses a laser measure to measure the distances from the wall to her couch at the angle she wants to be 8 ft. and 12 ft., but she does not have any way to mark the spots on the wall. How tall is the television that she wants?

2. Given the figure shown, find the height of the evergreen tree. Round your answers to the nearest thousandths.

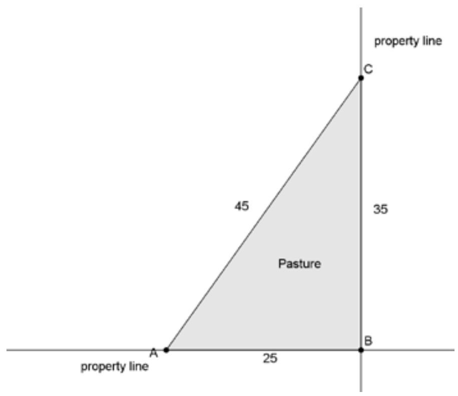


Name ____ Date ____________________

Lesson 10: Putting the Law of Cosines and the Law of Sines to Use

Exit Ticket

A triangular pasture is enclosed by fencing measuring 25, 35, and 45 yards at the corner of a farmer's property.



a. According to the fencing specifications, what is the measure of $\angle ABC$?

b. A survey of the land indicates that the property lines form a right angle at B. Explain why a portion of the pasture is actually on the neighboring property.

c. Where does the 45-yard section of the fence cross the vertical property line?

Name ____ Date ____________________

Lesson 11: Revisiting the Graphs of the Trigonometric Functions

Exit Ticket

Consider a sinusoidal function whose graph has an amplitude of 5, a period of $\frac{\pi}{2}$, a phase shift of $\frac{\pi}{4}$ units to the left, and a midline of $y = -3$. Write the function in the form $f(x) = A\sin\big(\omega(x-h)\big) + k$ for positive A, ω, h, k. Then, graph at least one full period of the function. Label the midline, amplitude, and period on the graph.

Name ____ Date ____________________

Lesson 12: Inverse Trigonometric Functions

Exit Ticket

1. State the domain and range for $f(x) = \sin^{-1}(x)$, $g(x) = \cos^{-1}(x)$, and $h(x) = \tan^{-1}(x)$.

2. Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Give answers in exact form.
   a. $2\sin(x) + \sqrt{3} = 0$

   b. $\tan^2(x) - 1 = 0$

3. Solve the trigonometric equation such that $0 \leq x \leq 2\pi$. Round to three decimal places.

$$\sqrt{5}\cos(x) - 2 = 0$$

Name ____ Date ____________________

Lesson 13: Modeling with Inverse Trigonometric Functions

Exit Ticket

The pedestal that the Statue of Liberty sits on is 89 ft. tall with a foundation fashioned in the shape of an eleven-point star making up the rest of the height. The front point of the star juts out about 145 ft. from the front of the statue and stands about 35.2 ft. tall.

How far from the Statue of Liberty does someone whose eye-height is 6 ft. need to stand in order to see the base of the statue without being obscured by the foundation? Include a diagram and appropriate work to justify your answer.

Name ______________________________ Date ____________

Lesson 14: Modeling with Inverse Trigonometric Functions

Exit Ticket

The minimum radius of the turn r needed for an aircraft traveling at true airspeed v is given by the following formula

$$r = \frac{v^2}{g\tan(\theta)}$$

where g is the acceleration due to gravity and θ is the banking angle of the aircraft. Use $g = 9.78\,\frac{\text{m}}{s^2}$ instead of $9.81\,\frac{\text{m}}{s^2}$ to model the acceleration of the airplane accurately at $30{,}000\text{ ft}$.

a. If an aircraft is traveling at $103\,\frac{\text{m}}{\text{s}}$, what banking angle is needed to successfully turn within 1 km?

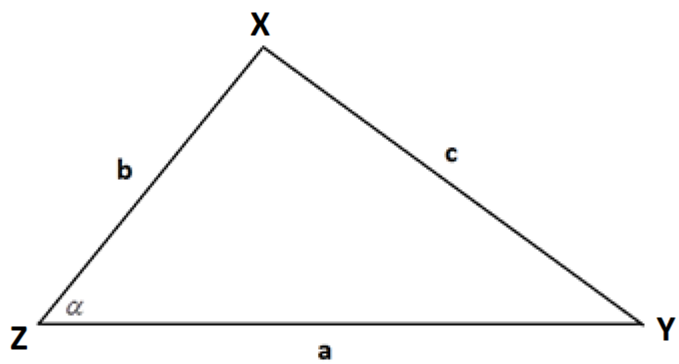
b. Write the formula that gives the banking angle as a function of the radius of the turn available for a fixed airspeed v.

c. For a variety of reasons, including motion sickness from fluctuating g-forces and the danger of losing lift, many airplanes have a maximum banking angle of around $60°$. Does this maximum on the model affect the domain or range of the formula you gave in part (b)?

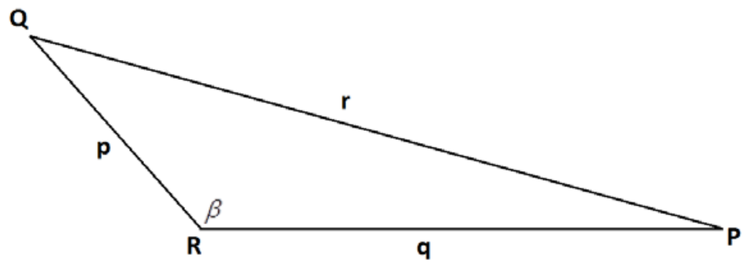
Name ______________________________________ Date ____________________

1.

a. In the following diagram triangle XYZ has side-lengths a, b, and c as shown. The angle α indicated is acute. Show that the area A of the triangle is given by $A = \frac{1}{2}ab\sin(\alpha)$.

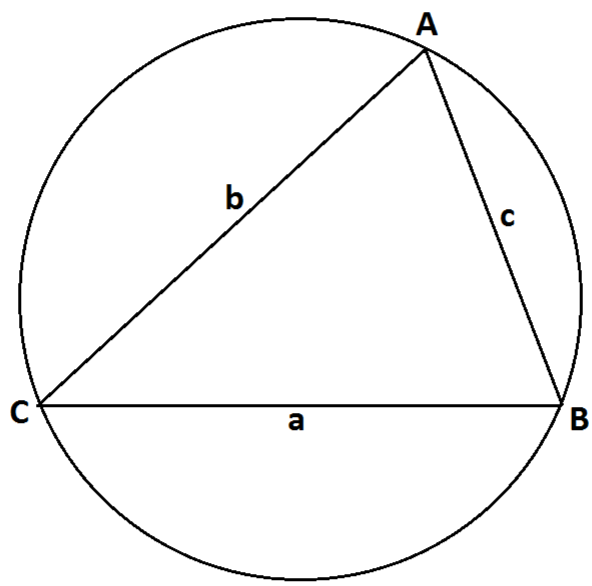


b. In the following diagram triangle PQR has side-lengths p, q, and r as shown. The angle β indicated is obtuse. Show that the area A of the triangle is given by $A = \frac{1}{2}pq\sin(\beta)$.



c. To one decimal place, what is the area of the triangle with sides of lengths 10 cm, 17 cm, and 21 cm? Explain how you obtain your answer.

2. Triangle ABC with side-lengths a, b, and c as shown is circumscribed by a circle with diameter d.



a. Show that $\frac{a}{\sin(A)} = d$.

b. The law of sines states that $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ for any triangle ABC with side lengths a, b, and c (with the side of length a opposite vertex A, the side of length b opposite vertex B, and the side of length c opposite vertex C). Explain why the law of sines holds for all triangles.

c. Prove that $c^2 = a^2 + b^2 - 2ab\cos(C)$ for the triangle shown in the original diagram.

3. Beatrice is standing 20 meters directly east of Ari, and Cece is standing 15 meters directly north-east of Beatrice.

   a. To one decimal place, what is the distance between Ari and Cece?

   b. To one decimal place, what is the measure of the smallest angle in the triangle formed by Ari, Beatrice, and Cece?

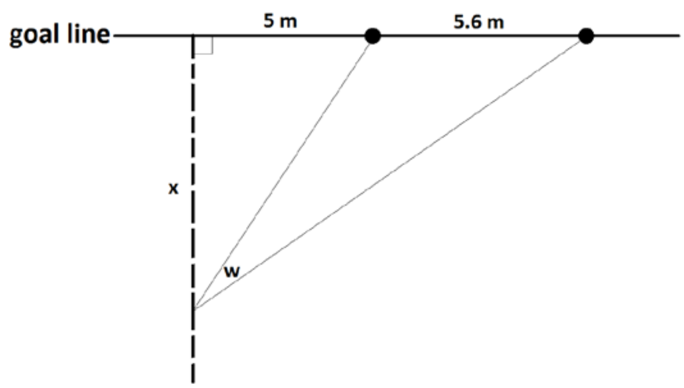
4.

a. Is it possible to construct an inverse to the sine function if the domain of the sine function is restricted to the set of real values between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$? If so, what is the value of $\sin^{-1}\left(\frac{1}{2}\right)$ for this inverse function? Explain how you reach your conclusions.

b. Is it possible to construct an inverse to the cosine function if the domain of the cosine function is restricted to the set of real values between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$? If so, what is the value of $\cos^{-1}\left(-\frac{1}{2}\right)$ for this inverse function? Explain how you reach your conclusions.

c. Is it possible to construct an inverse to the tangent function if the domain of the tangent function is restricted to the set of real values between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$? If so, what is the value of $\tan^{-1}(-1)$ for this inverse function? Explain how you reach your conclusions.

5. The diagram shows part of a rugby union football field. The goal line (marked) passes through two goal posts (marked as black circles) set 5.6 meters apart.

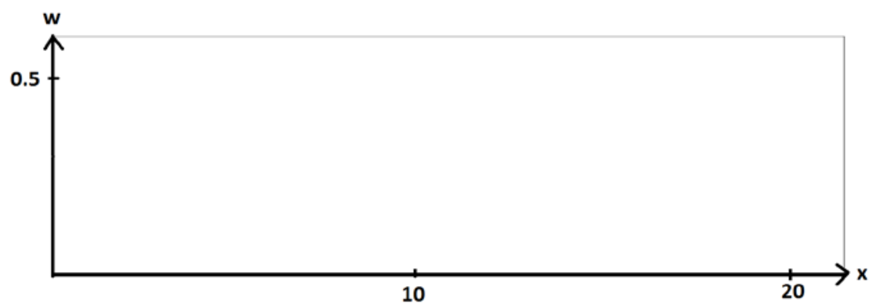


According to the rules of the game, an attempt at a conversion must be taken at a point on a line through the point of touchdown and perpendicular to the goal line. If a touchdown occurred 5 meters to one side of a goal post on the goal line, for example, the dashed line in the diagram indicates the line on which the conversion must be attempted.

Suppose the conversion is attempted at a distance of x meters from the goal line. Let w be the angle (measured in radians) indicated subtended by the goal posts.

a. Using inverse trigonometric functions, write an expression for w in terms of the distance x.

b. Using a graphing calculator or mathematics software, sketch a copy of the graph of the angle measure w as a function of x on the axes below. Indicate on your sketch the value of x that maximizes w. What is that maximal angle measure? (Give all your answers to two decimal places.)



c. In the original diagram, we see that the angle of measure w is one of three angles in an obtuse triangle. To two decimal places, what is the measure of the obtuse angle in that triangle when w has its maximal possible measure?

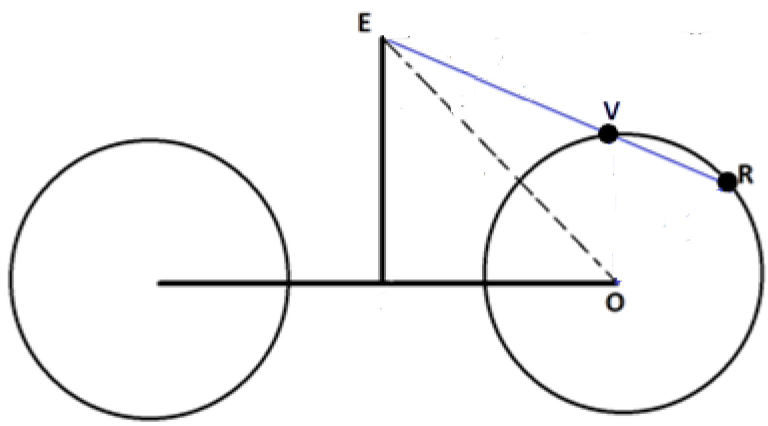
6. While riding her bicycle, Anu looks down for an instant to notice a reflector attached to the front wheel near its rim. As the bicycle moves, the wheel rotates and the position of the reflector relative to the frame of the bicycle changes. Consequently, the angle down from the horizontal that Anu needs to look in order to see the reflector changes with time.

   Anu also notices the air-valve on the rim of the front wheel tire and observes that the valve and the reflector mark off about one-sixth of the perimeter of the front wheel.

   As Anu rides along a straight path, she knows that there will be a moment in time when the reflector, the valve, and her eye will be in line. She wonders what the angle between the horizontal from her eye and the line from her eye to the reflector passing through the valve is at this special moment.

   She estimates that the reflector and the valve are each 1.5 feet from the center of the front wheel, that her eye is 6 feet away from that center of that wheel, and that the line between her eye and the wheel center is $45°$ down from the horizontal.

   According to these estimates what is the measure, to one decimal place in radians, of the angle Anu seeks?



In this diagram, O represents the center of the front wheel, E the location of Anu's eye, and R and V the positions of the reflector and valve, respectively, at the instant R, V, and E are collinear.